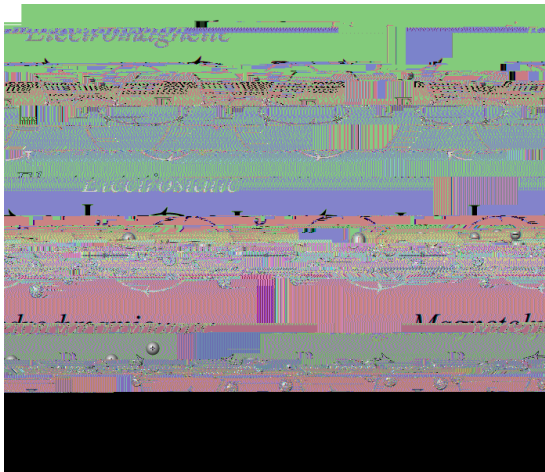
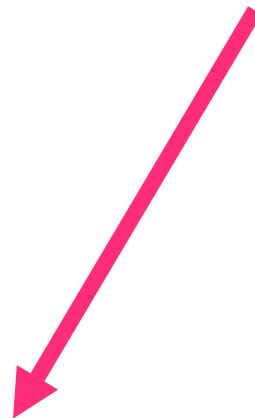
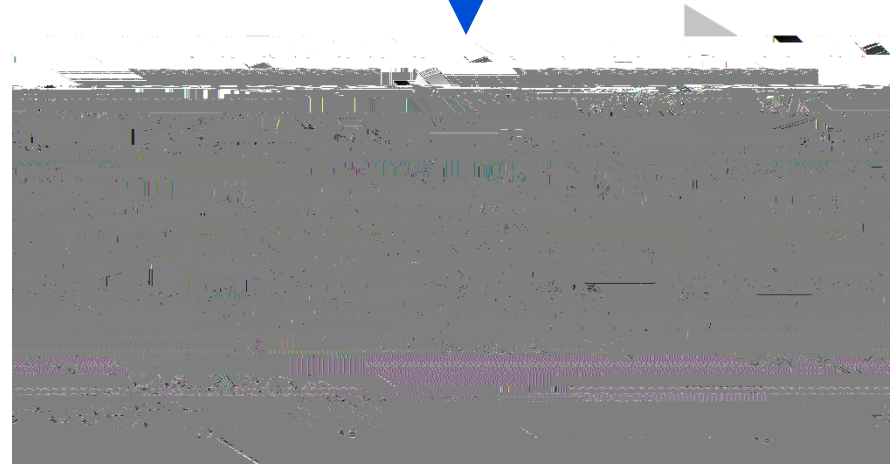
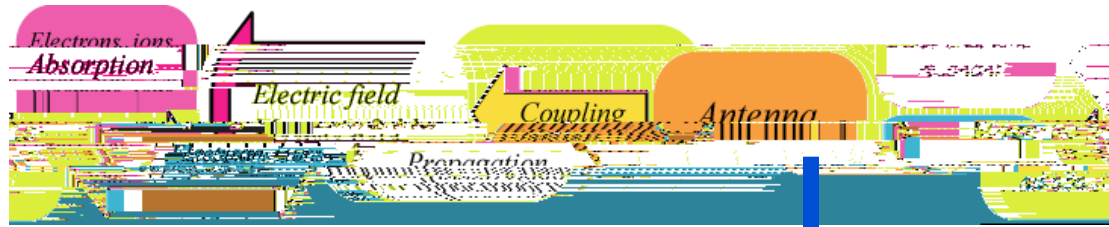
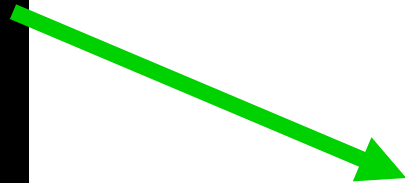
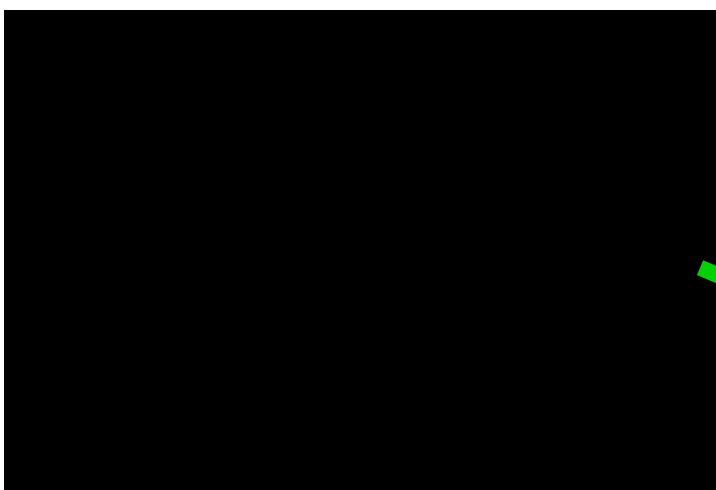


Physics of Landau and Cyclotron Resonances :

- Active and reactive power
- Plasma resonances
- Resonant interaction
- Random phase approximation RPA
- Quasi linear equation
- Landau absorption
- Cyclotron absorption
- Current generation 1D
- Current generation 2D
- Free energy extraction







Reactive power exchange

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m} \mathbf{E} \exp(-j\omega t) - \mathbf{v} \times \mathbf{B}$$

Active power exchange

$$\neq 0 \quad \frac{d\mathbf{v}}{dt} = \frac{q}{m} \mathbf{E} \exp(-j\omega t) - \mathbf{v} \times \mathbf{B}$$

$$k_{\parallel} v_{\parallel} - \omega \ll 0$$

$$k_{\parallel} v_{\parallel} - \omega \gg 0$$

Resonant collisionless power exchange

$$k_{\parallel} v_{\parallel} - \omega \approx 0$$

$$\frac{dv_{\parallel}}{dt} = \frac{q}{m} E \cos(\omega t - k_{\parallel} v_{\parallel} t)$$

$$\frac{dv_{\parallel}}{dt} = \frac{q}{m} E \cos(\omega t - k_{\parallel} v_{\parallel} t)$$



Cold Plasma Resonances

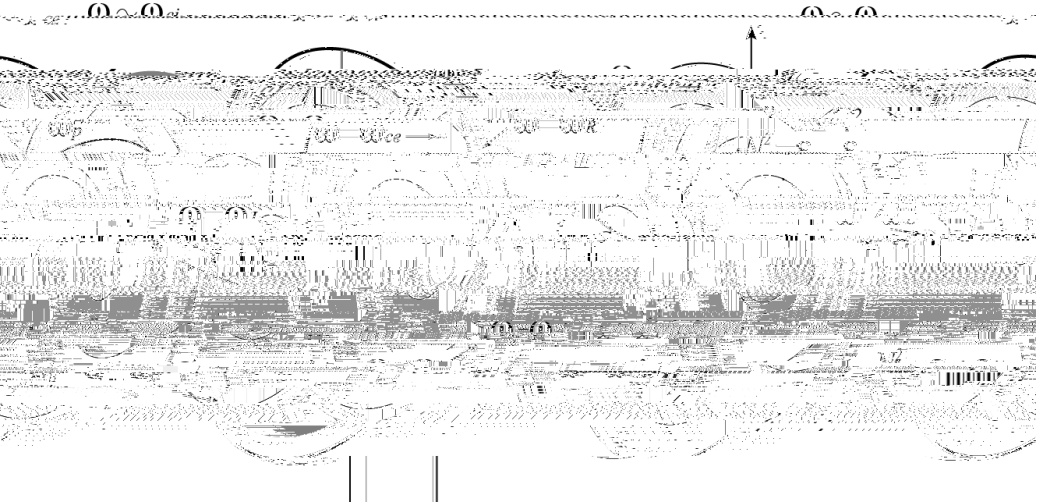
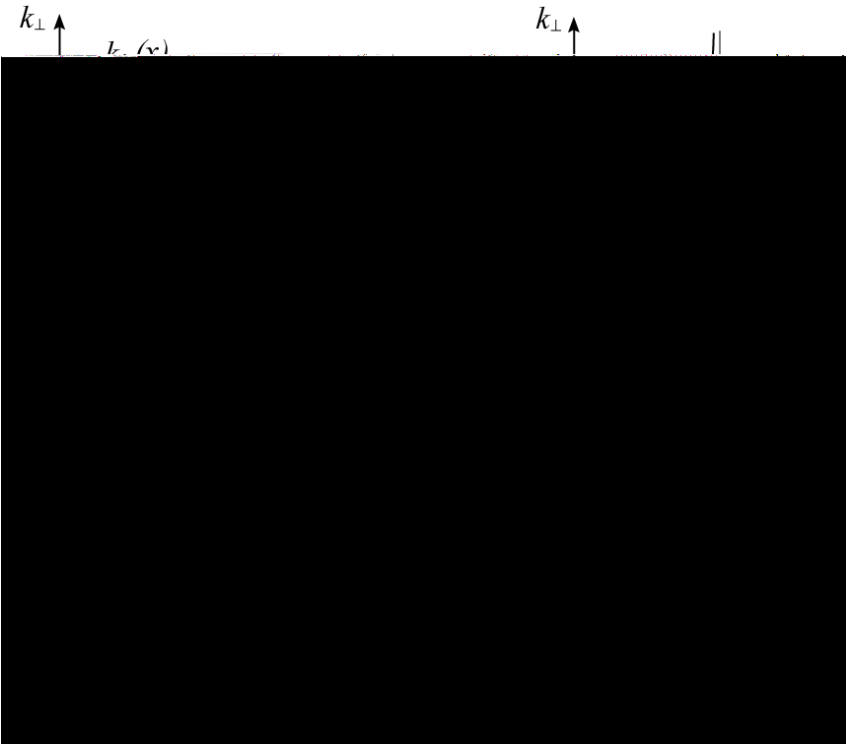
$$\begin{vmatrix} \epsilon_{\perp} - N_{\parallel}^2 & -j\epsilon_{\times} & N_{\perp}N_{\parallel} \\ j\epsilon_{\times} & \epsilon_{\perp} - N_{\parallel}^2 - N_{\perp}^2 & 0 \\ N_{\perp}N_{\parallel} & 0 & \epsilon_{\parallel} - N_{\parallel}^2 \end{vmatrix} = 0$$

$$\begin{vmatrix} \epsilon_{\perp} - N_{\parallel}^2 & -j\epsilon_{\times} & N_{\perp}N_{\parallel} \\ j\epsilon_{\times} & \epsilon_{\perp} - N_{\parallel}^2 - N_{\perp}^2 & 0 \\ N_{\perp}N_{\parallel} & 0 & \epsilon_{\parallel} - N_{\parallel}^2 \end{vmatrix} = 0$$

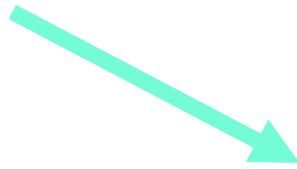
$$\begin{vmatrix} \epsilon_{\perp} - N_{\parallel}^2 & -j\epsilon_{\times} \\ j\epsilon_{\times} & \epsilon_{\perp} - N_{\parallel}^2 - N_{\perp}^2 \end{vmatrix} = 0$$

$$b = \frac{(\epsilon_{\perp} + \epsilon_{\parallel}) (\epsilon_{\perp} - N_{\parallel}^2) - \epsilon_{\times}^2}{(\epsilon_{\perp} - N_{\parallel}^2) (\epsilon_{\parallel} - N_{\parallel}^2)}$$

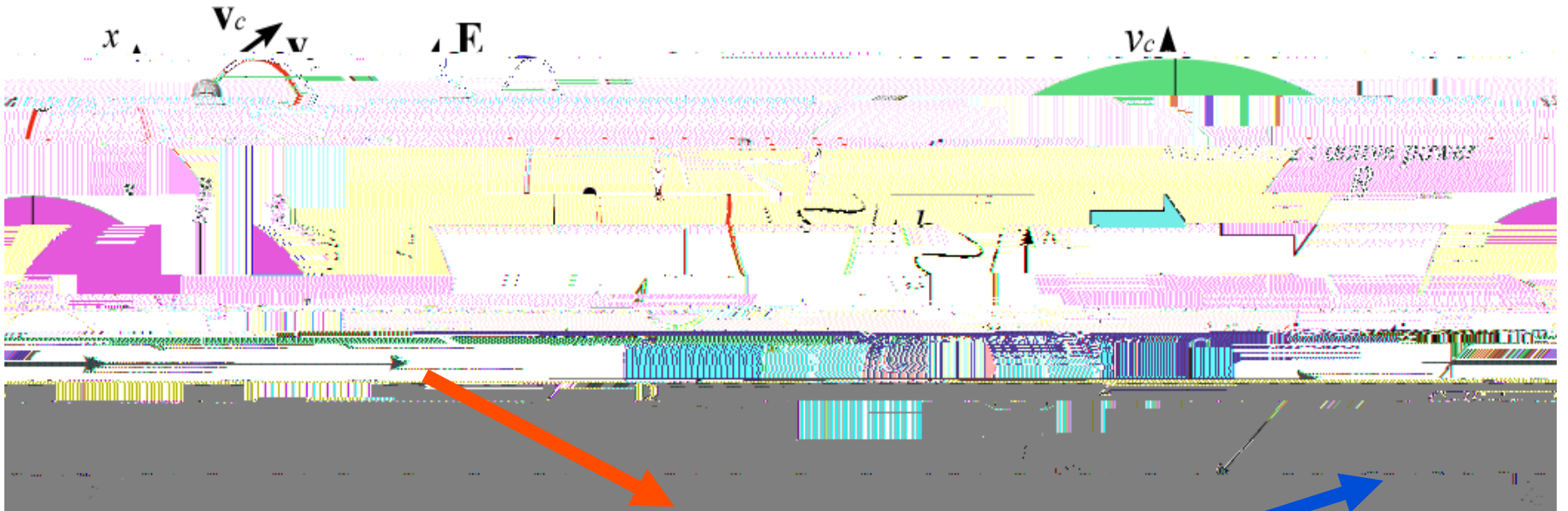
$\epsilon_{\perp} - N_{\parallel}^2 = 0$ Resonances $\epsilon_{\parallel} - N_{\parallel}^2 = 0$
 convergence of active power localisation of reactive power
 convergence of active power localisation of reactive power



Resonant Absorption



Waves - Particles Resonances



Landau Resonances

$$\omega = k_{\parallel} v_{\parallel}$$

Cyclotron Resonances

$$\omega - k_{\parallel} v_{\parallel} = n\omega_c$$

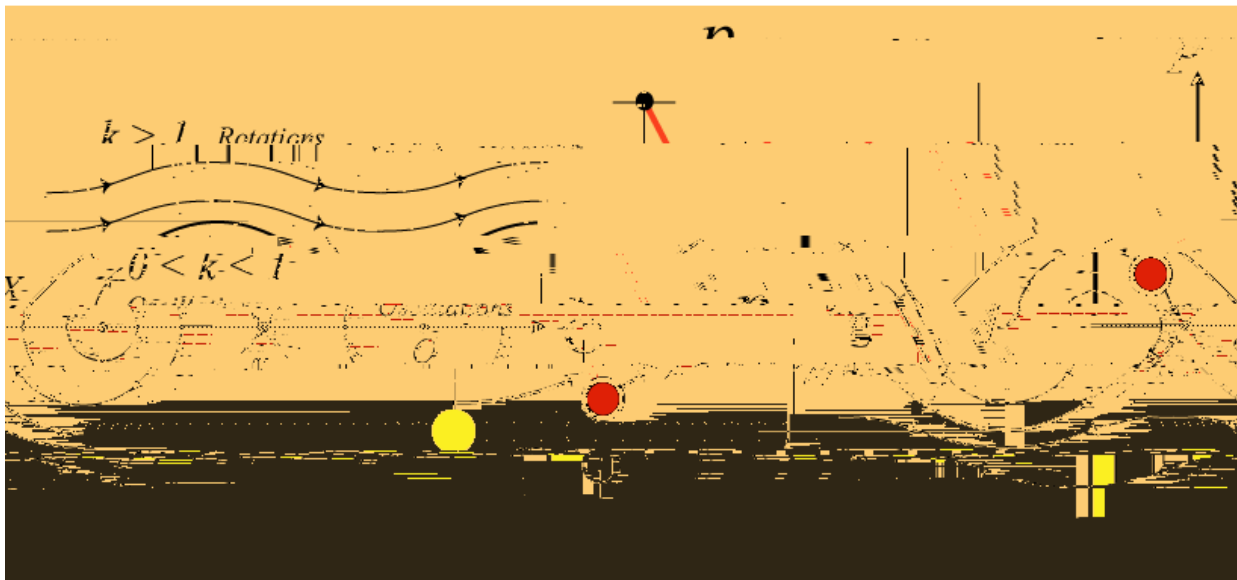




$$\frac{d\varphi}{dt} = I - \omega_0$$

Potential Energy $\propto \Omega^2 \cos \varphi$ Kinetic Energy $\propto (I - \omega_0)^2$

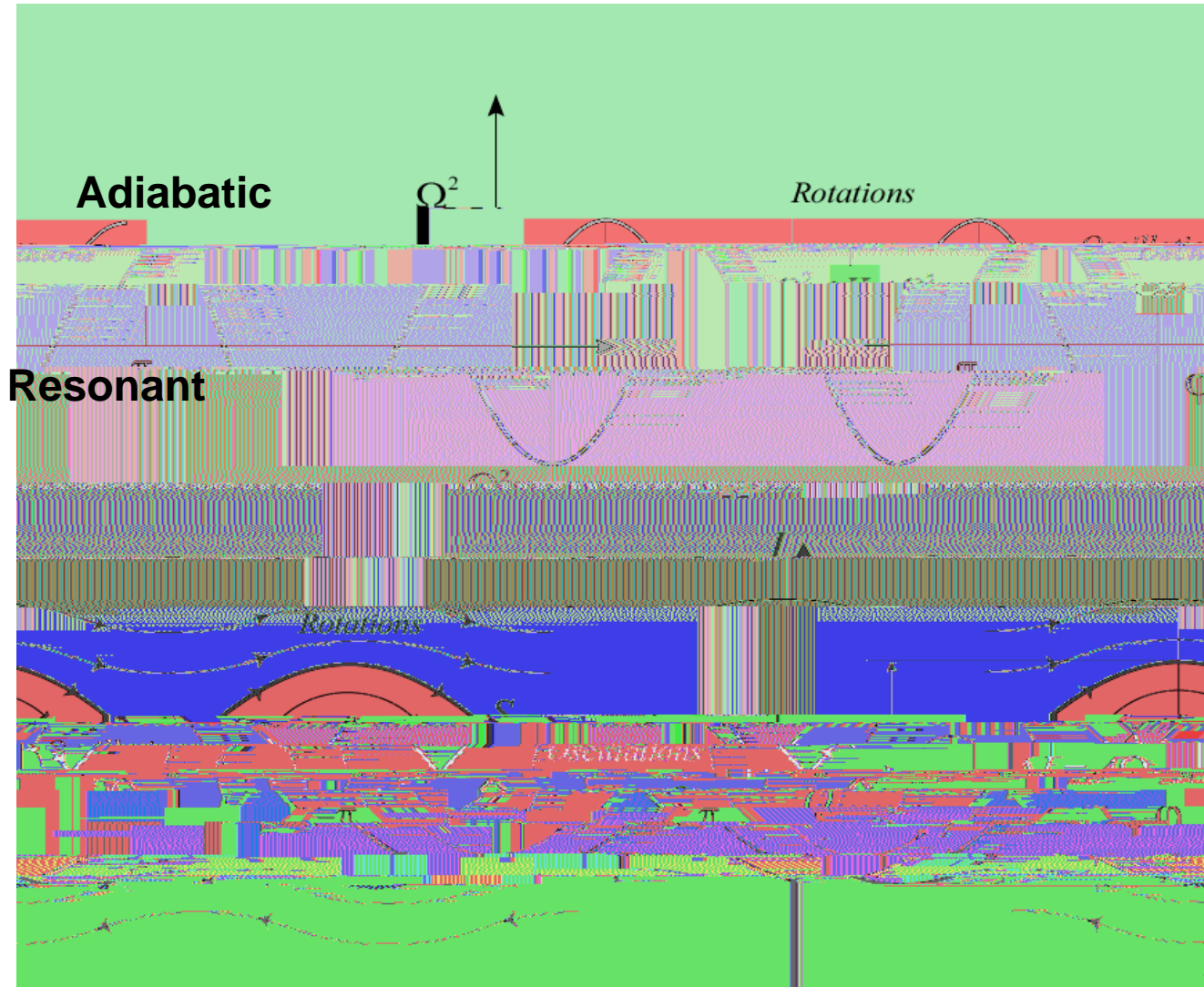
Nonlinear pendulum

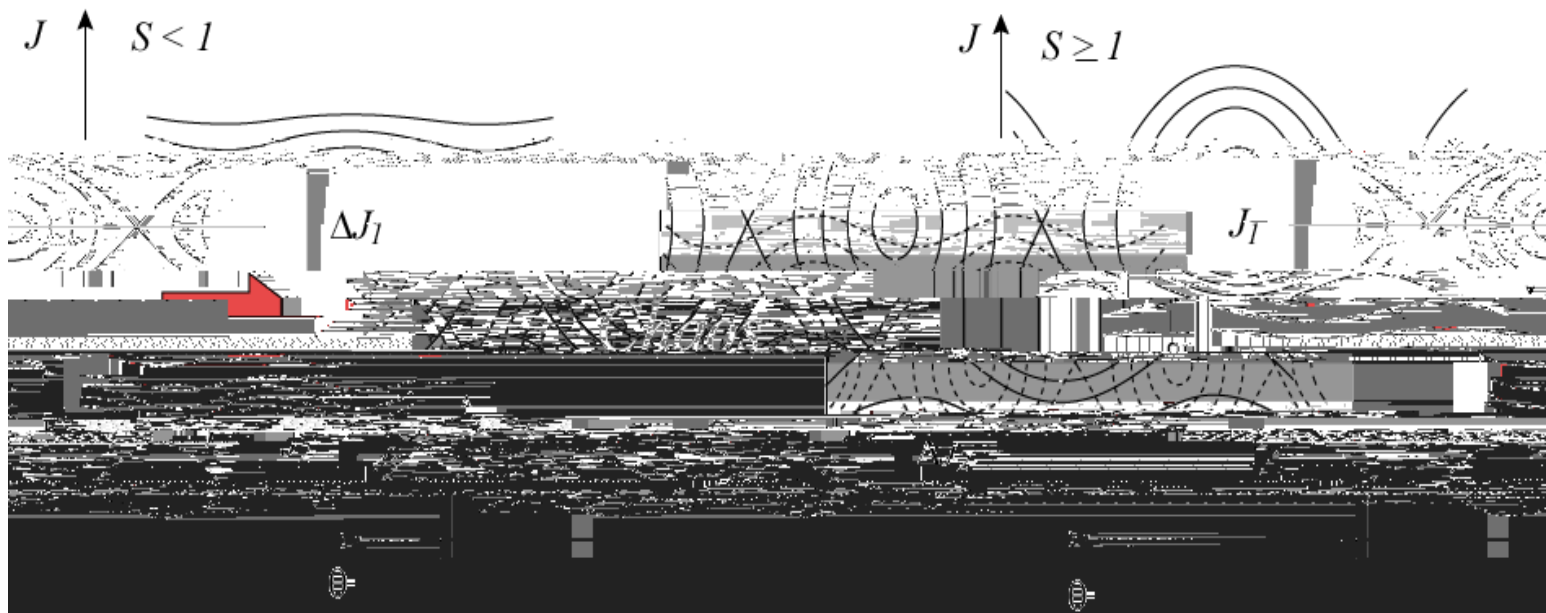


Nonlinear pendulum



$$\text{Potential } \cdot E = \Omega^2 \cos \varphi \quad \text{Kinetic } \cdot E = \frac{(I - \omega_0)^2}{2}$$





Resonance



Resonances



Chaos



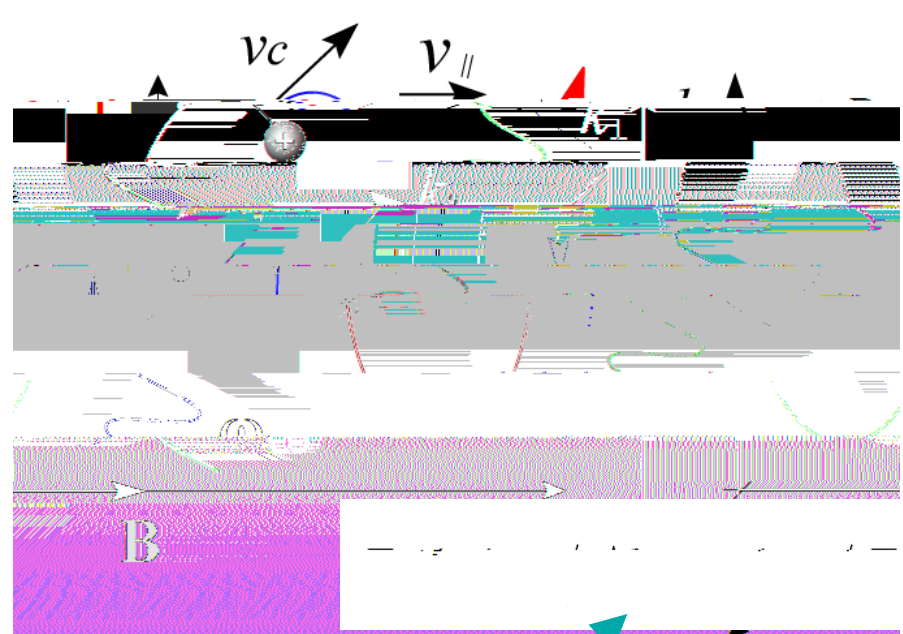
Random Walk

RPA

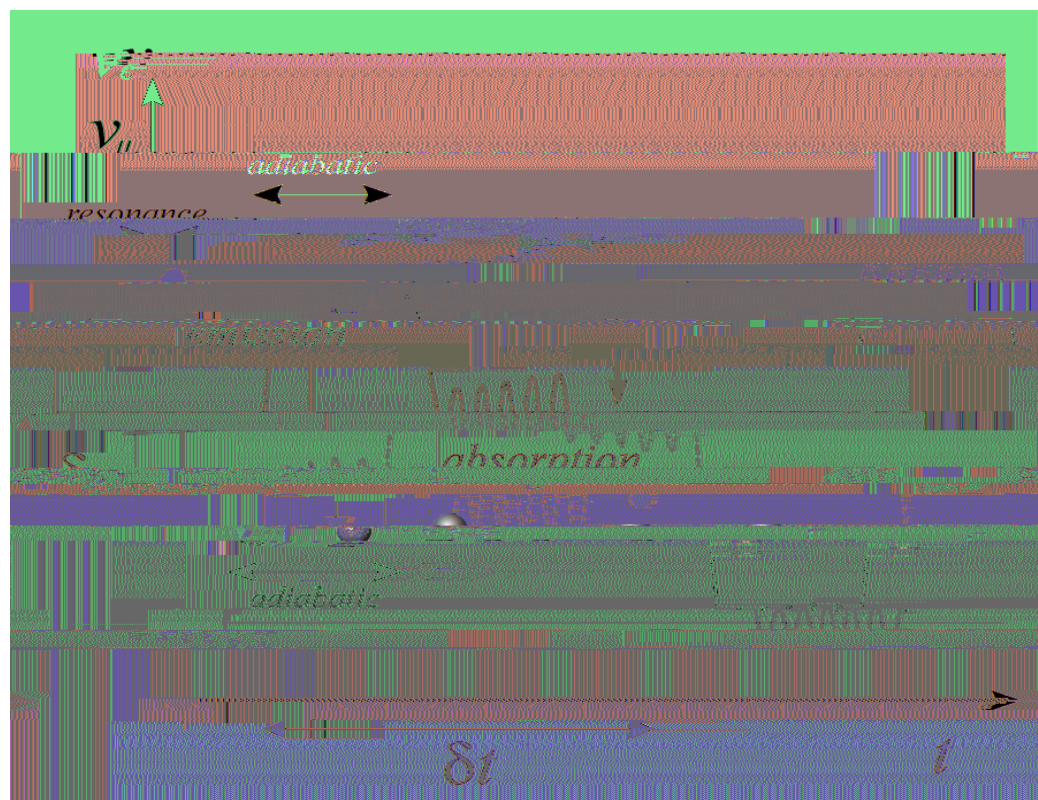


Quasilinear theory





RPA



Quasi linear equation



$$\frac{\partial F(\mathbf{v}, t)}{\partial t} = \int_V [m(\mathbf{v} \leftarrow \mathbf{v} + \mathbf{x}) E(\mathbf{v} + \mathbf{x}, t) - m(\mathbf{v} + \mathbf{x} \leftarrow \mathbf{v}) E(\mathbf{x}, t)] d\mathbf{x}$$

$$\frac{\partial F(\mathbf{v}, t)}{\partial t} = \int [m(\mathbf{v} \leftarrow \mathbf{v} + \mathbf{x}) E(\mathbf{v} + \mathbf{x}, t) - m(\mathbf{v} + \mathbf{x} \leftarrow \mathbf{v}) E(\mathbf{x}, t)] d\mathbf{x}$$



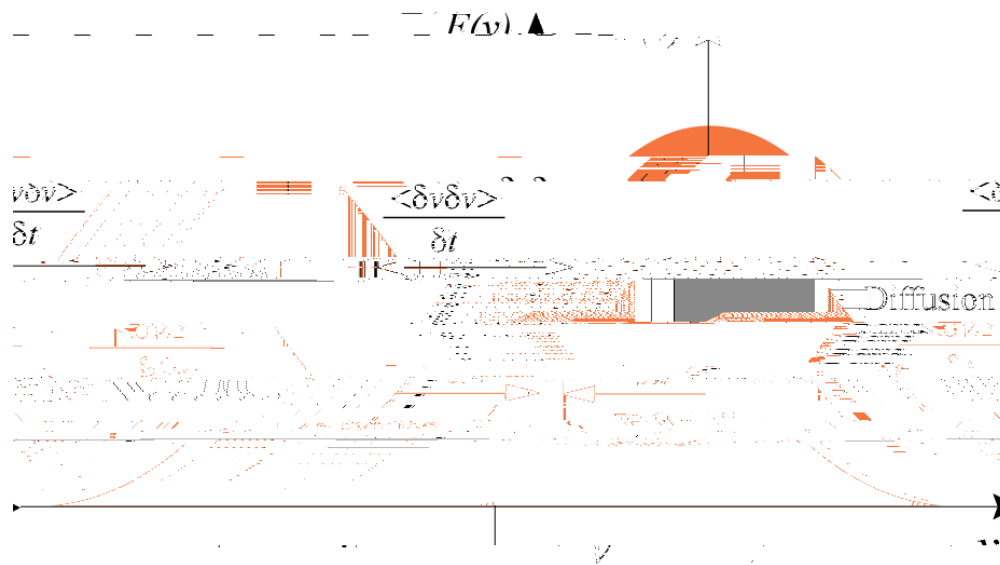
$$\frac{\partial F(\mathbf{v}, t)}{\partial t} = \int [w(\mathbf{v} \leftarrow \mathbf{v} + \mathbf{x}) F(\mathbf{v} + \mathbf{x}, t) - w(\mathbf{v} + \mathbf{x} \leftarrow \mathbf{v}) F(\mathbf{v}, t)] d\mathbf{x}$$

Taylor : Kramer-Moyal

$$w(\mathbf{v} \leftarrow \mathbf{v} + \mathbf{x}) F(\mathbf{v} + \mathbf{x}) = w(\mathbf{v} - \mathbf{x} \leftarrow \mathbf{v}) F(\mathbf{v}) + \dots$$

$$\int d\mathbf{x} [w(\mathbf{v} - \mathbf{x} \leftarrow \mathbf{v}) - w(\mathbf{v} + \mathbf{x} \leftarrow \mathbf{v})] = 0$$





Microreversibility : $w(\mathbf{v} + \mathbf{x} \leftarrow \mathbf{v}) = w(\mathbf{v} \leftarrow \mathbf{v} + \mathbf{x})$

$$\frac{\partial w(\mathbf{v} + \mathbf{x} \leftarrow \mathbf{v})}{\partial t} = w(\mathbf{v} \leftarrow \mathbf{v} + \mathbf{x}) - w(\mathbf{v} + \mathbf{x} \leftarrow \mathbf{v}) = w(\mathbf{v} \leftarrow \mathbf{v} + \mathbf{x}) - w(\mathbf{v} + \mathbf{x} \leftarrow \mathbf{v})$$

$$\frac{1}{2} \int \mathbf{x} w(\mathbf{v} + \mathbf{x} \leftarrow \mathbf{v}) d\mathbf{x} - \frac{1}{2} \int \mathbf{x} w(\mathbf{v} - \mathbf{x} \leftarrow \mathbf{v}) d\mathbf{x} = \frac{\langle \delta \mathbf{v} \rangle}{\delta t} = \int \mathbf{x} \frac{\partial w(\mathbf{v} - \mathbf{x} \leftarrow \mathbf{v})}{\partial t} d\mathbf{x}$$



$$\text{Einstein Relation: } \frac{\partial}{\partial t} \cdot \frac{\langle \delta \mathbf{v} \delta \mathbf{v} \rangle}{\langle \delta \mathbf{v} \rangle} = \frac{\langle \delta \mathbf{v} \rangle}{\langle \delta \mathbf{v} \rangle}$$

$$\frac{\partial E}{\partial t} = \frac{\partial \langle \delta \mathbf{v} \delta \mathbf{v} \rangle}{\partial t} = \frac{\partial E}{\partial t}$$



How to quantify the impact of the geometry on the RPA dispersion curves?

$\theta = 1$

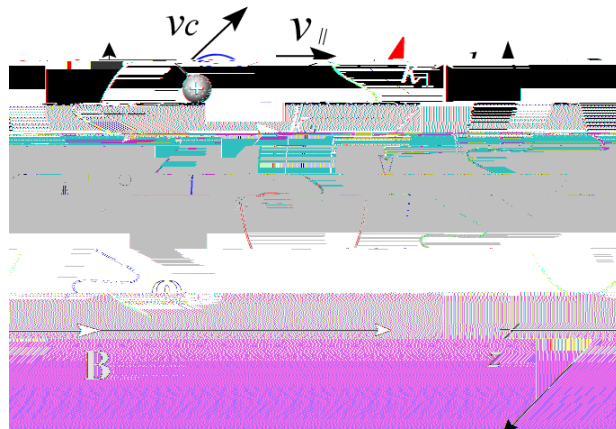
$\theta = 10^\circ$

*

+

$\theta = 10^\circ$

$\theta = 1$



RPA





$$M \dots dv_{\parallel} \dots q d \dots$$

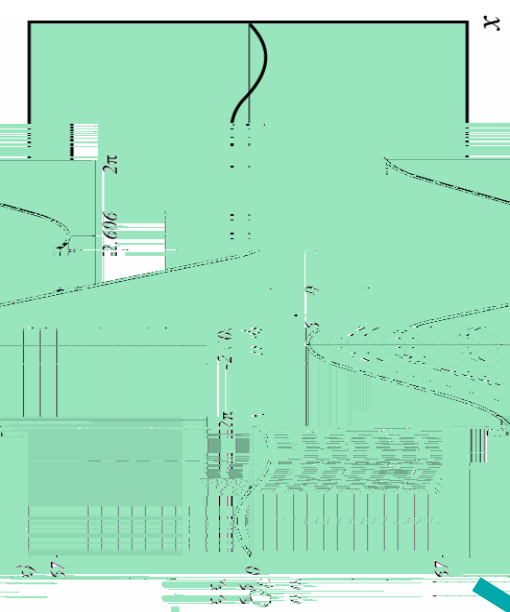
$$v' = v_{\parallel} - \omega/k_{\parallel}$$



$$v' = v_{\parallel} - \omega/k_{\parallel}$$



$$\sin^2(m/r) / \omega^2 \dots$$



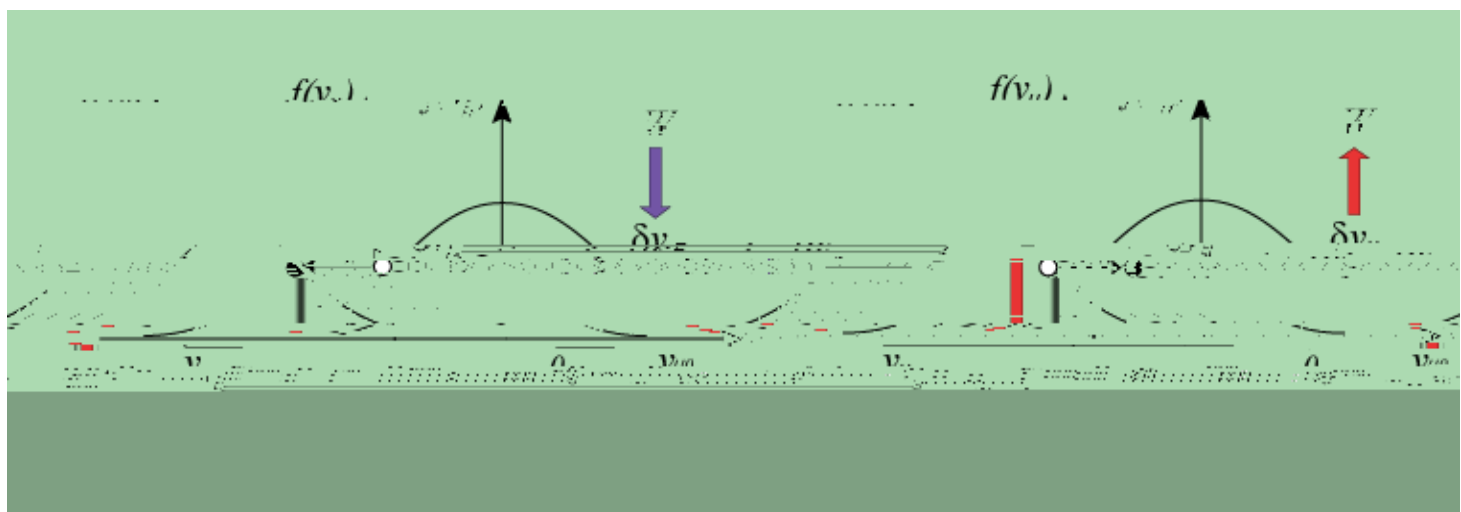
$$k_{\parallel} v_{\parallel} - \omega \gg 0$$



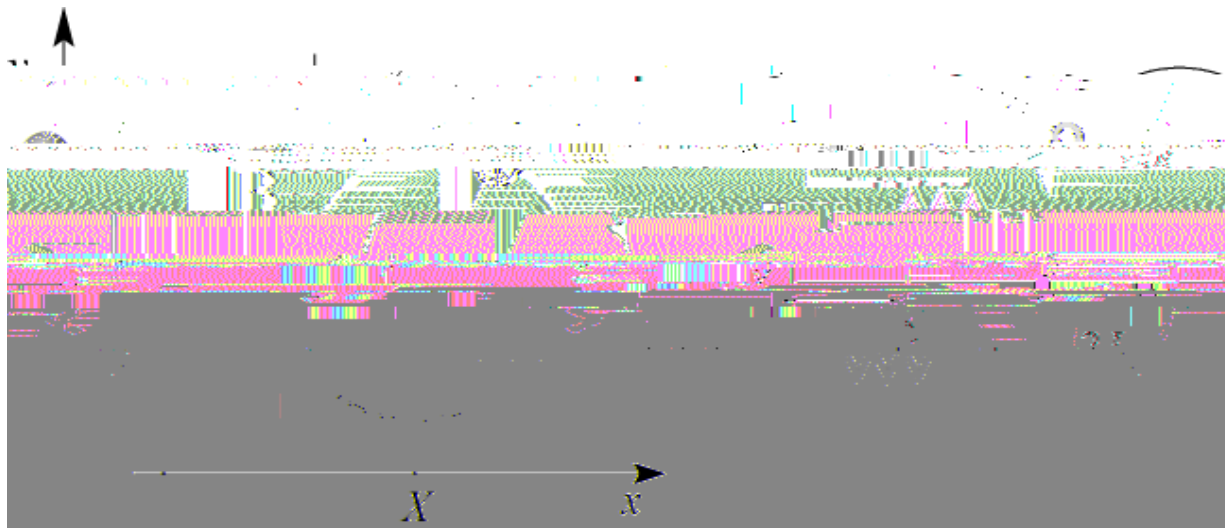
$$k_{\parallel} v_{\parallel} - \omega \approx 0$$

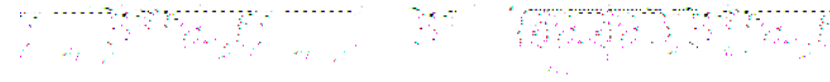
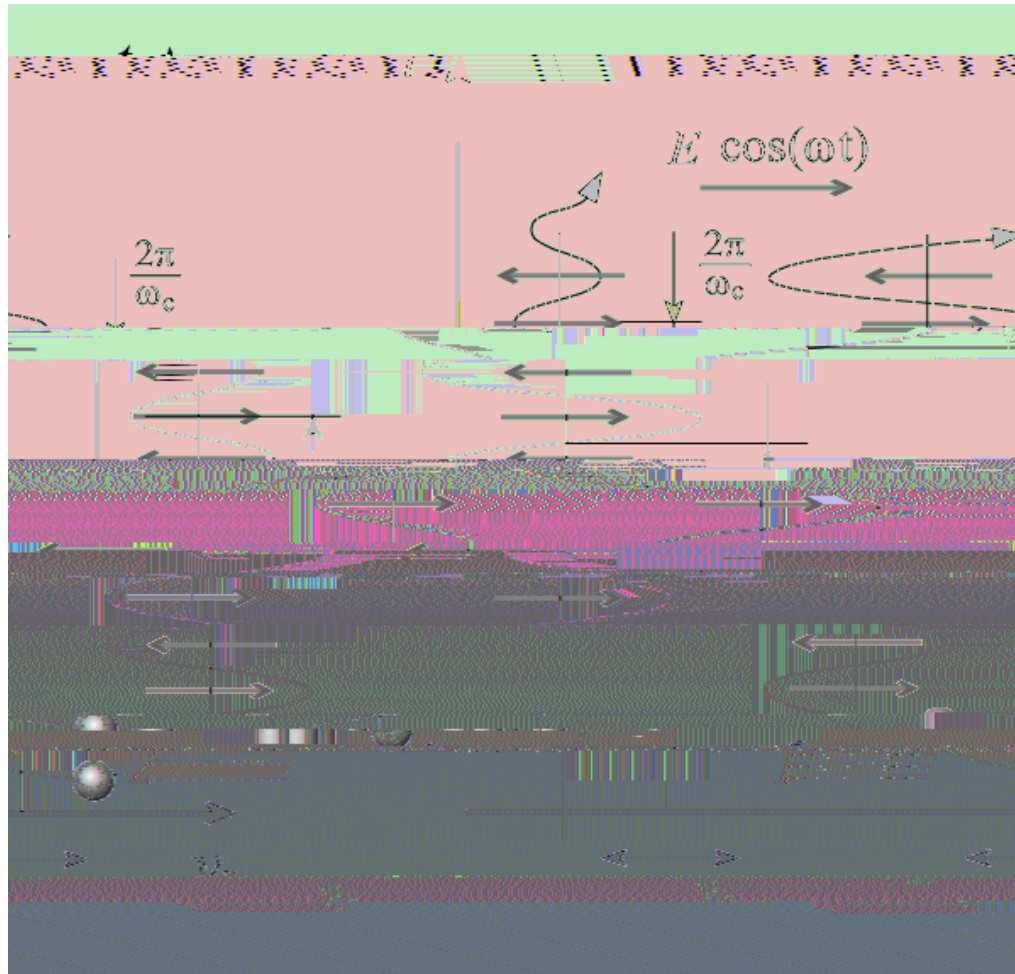
$$k_{\parallel} v_{\parallel} - \omega \ll 0$$

$$c \sin^2(\theta) / \alpha^2 | \pi \delta(\omega) |$$

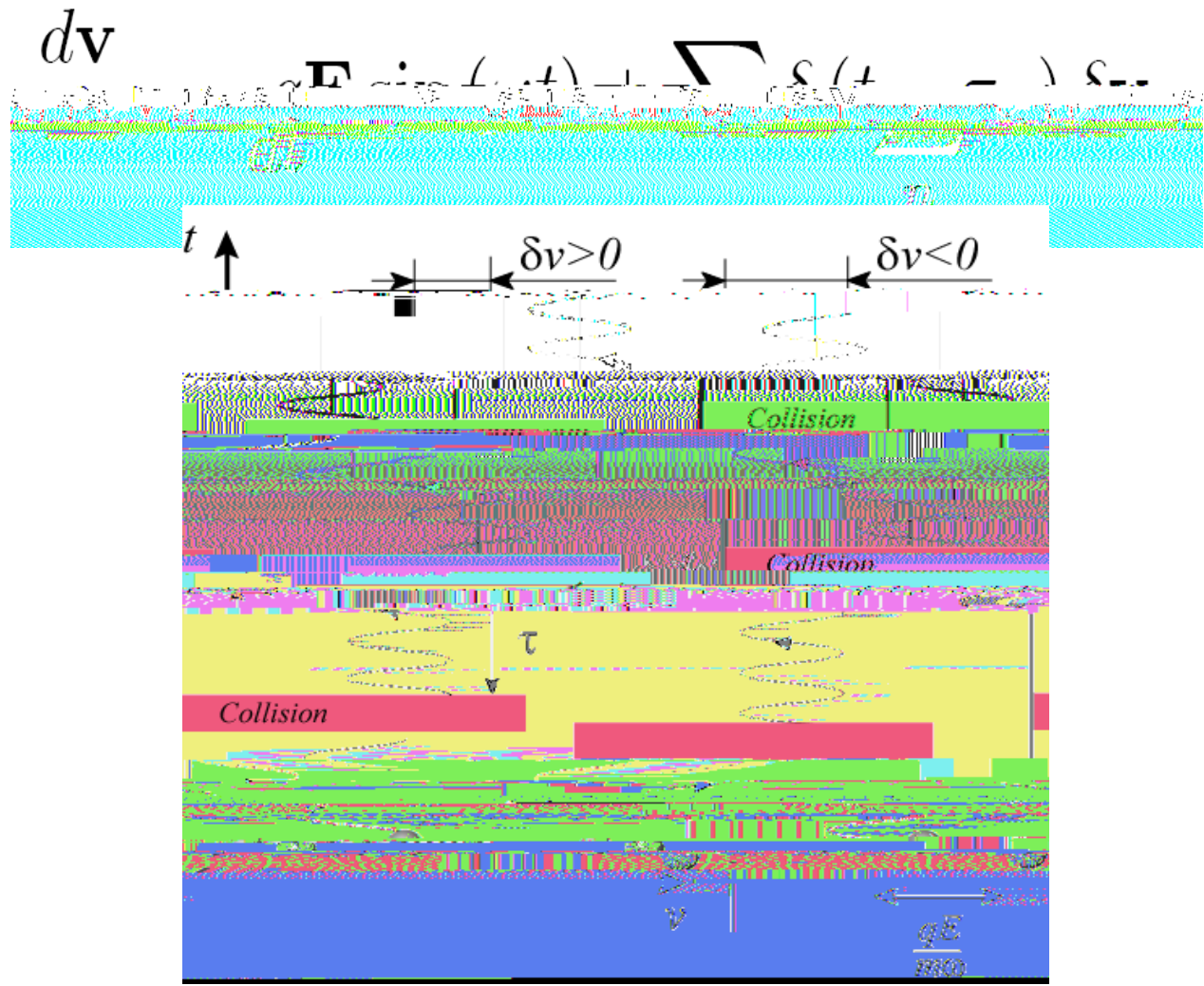


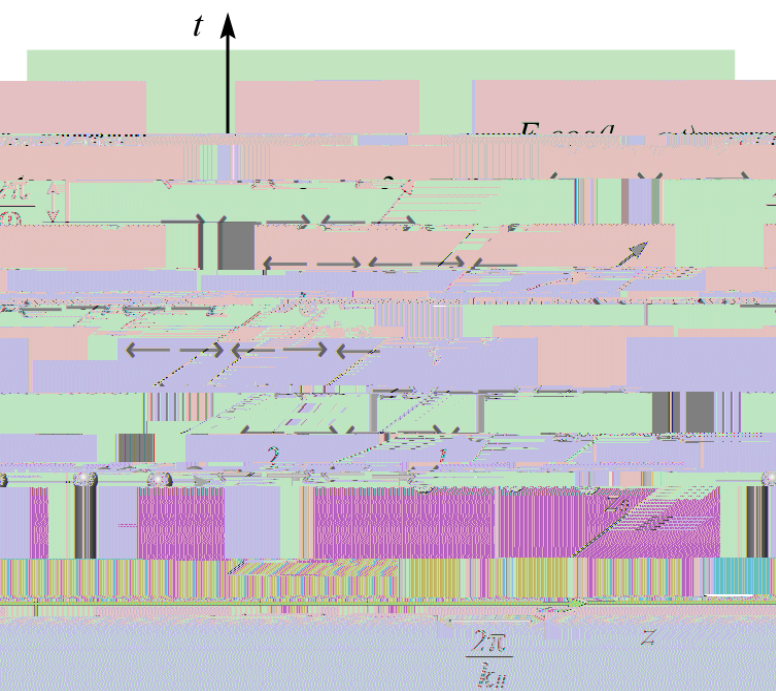
Cyclotron absorption



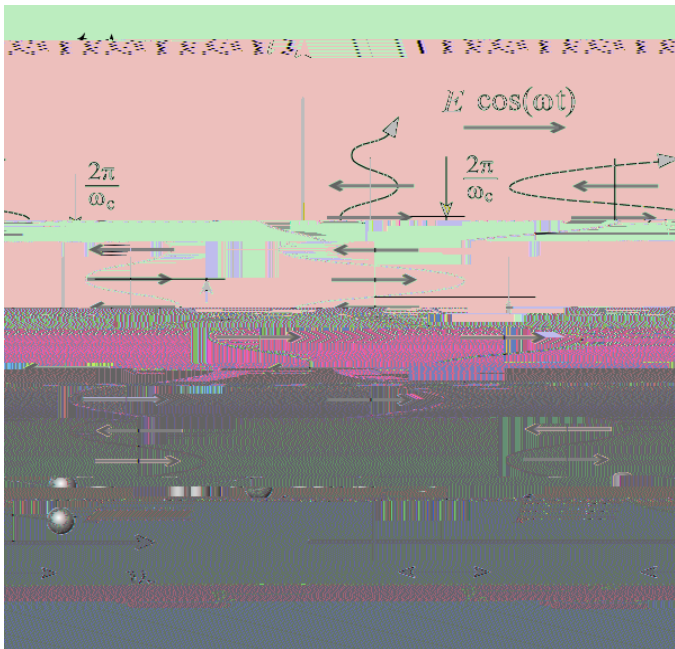


Inverse Bremsstrahlung

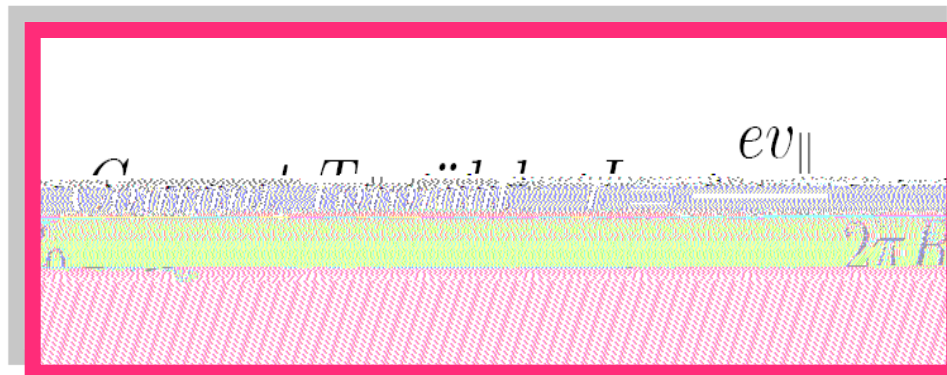
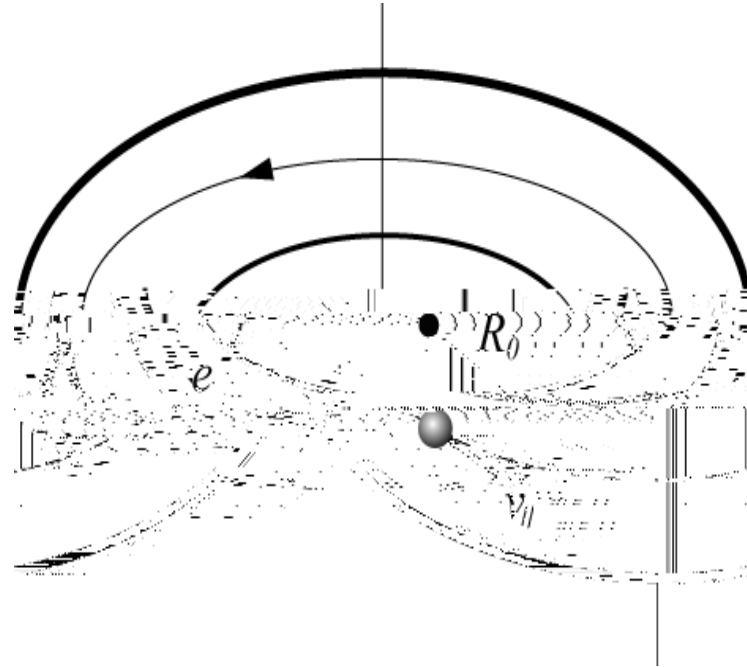




Finite Larmor radius effect :



Current Generation

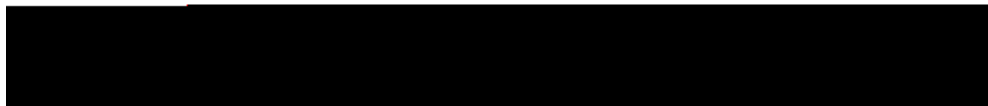


Thermoélectric effect

Spitzer Conductivity

$$\mu_{\parallel} = \left| \frac{e}{m_e \nu_e} \right| \quad \longrightarrow$$

$$T^{\frac{3}{2}}$$

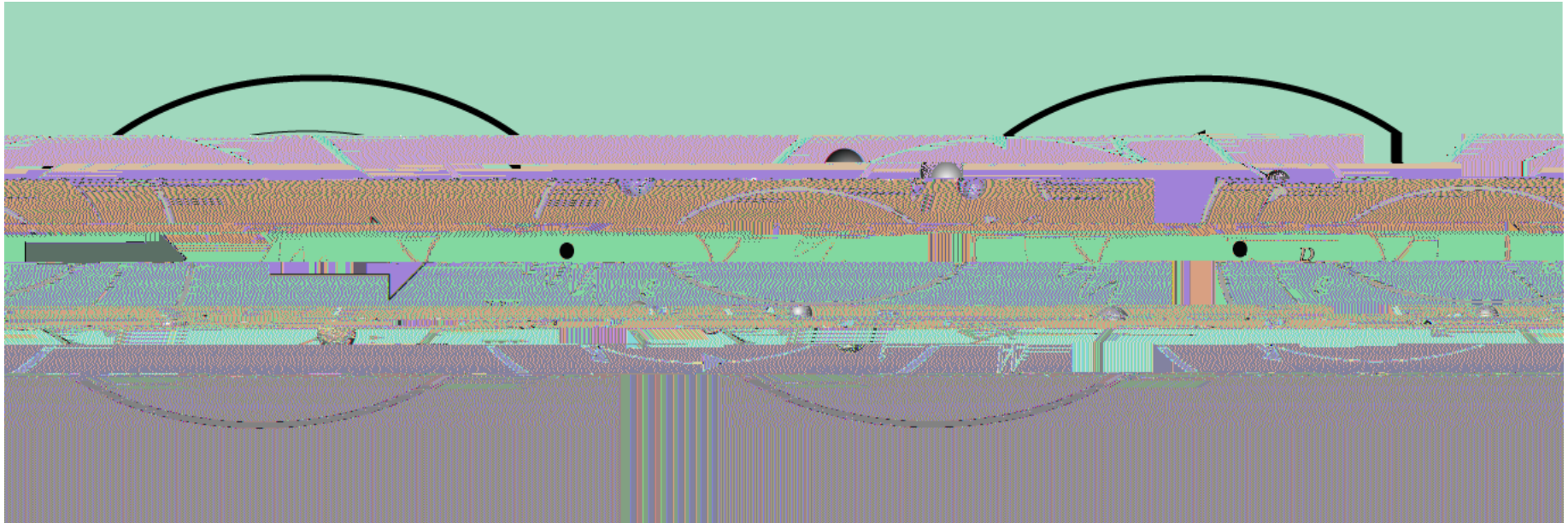


$$\int_{\Omega} \left(\frac{\partial n_e}{\partial t} \right) ds$$

$$\int_{\Omega} \left(\frac{\partial n_e}{\partial t} \right) ds$$



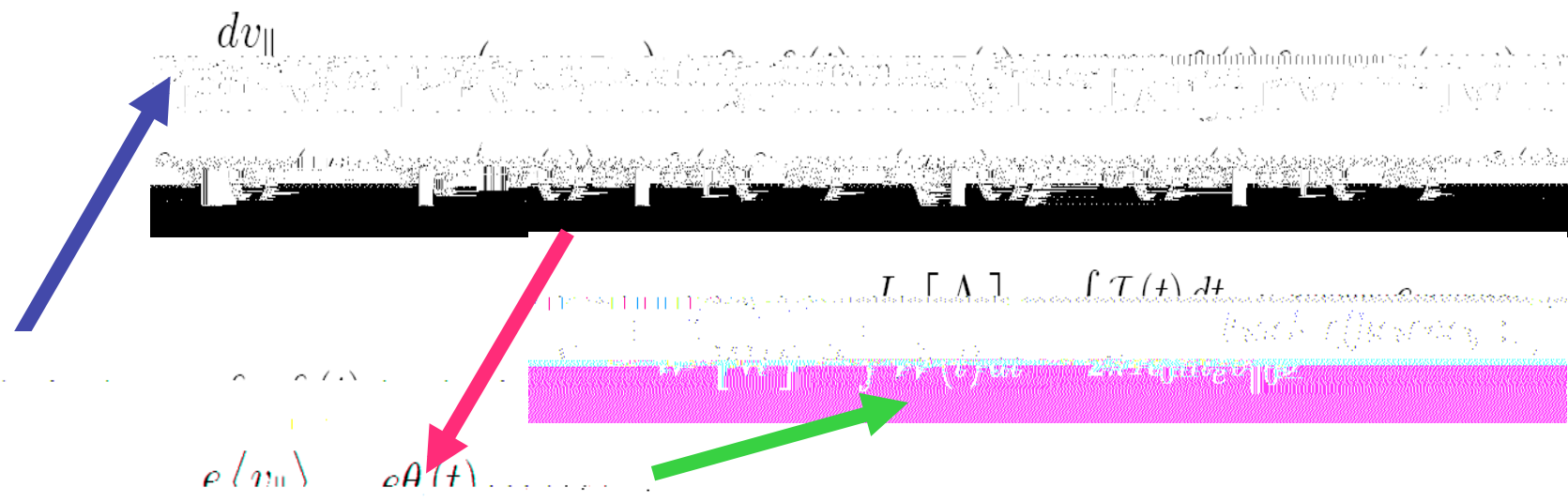
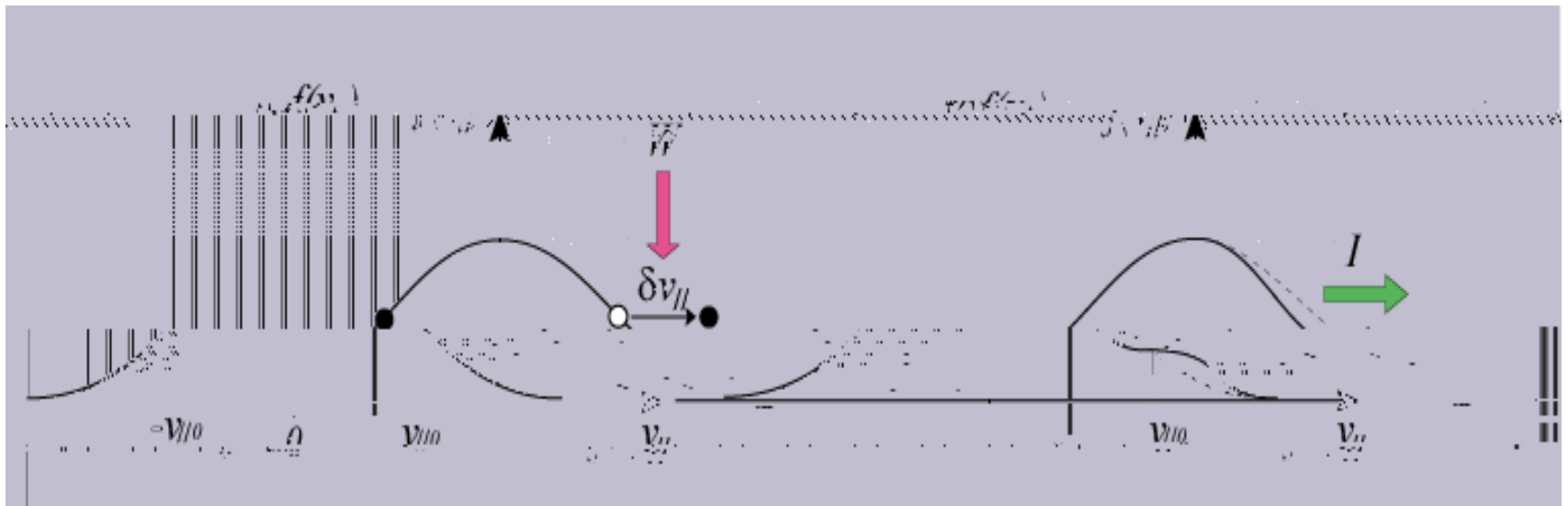
$$\oint_{\partial \Omega} T_e^{\frac{5}{2}} \nabla_{\parallel} \ln n_e ds \approx 5T_e^{\frac{5}{2}}$$

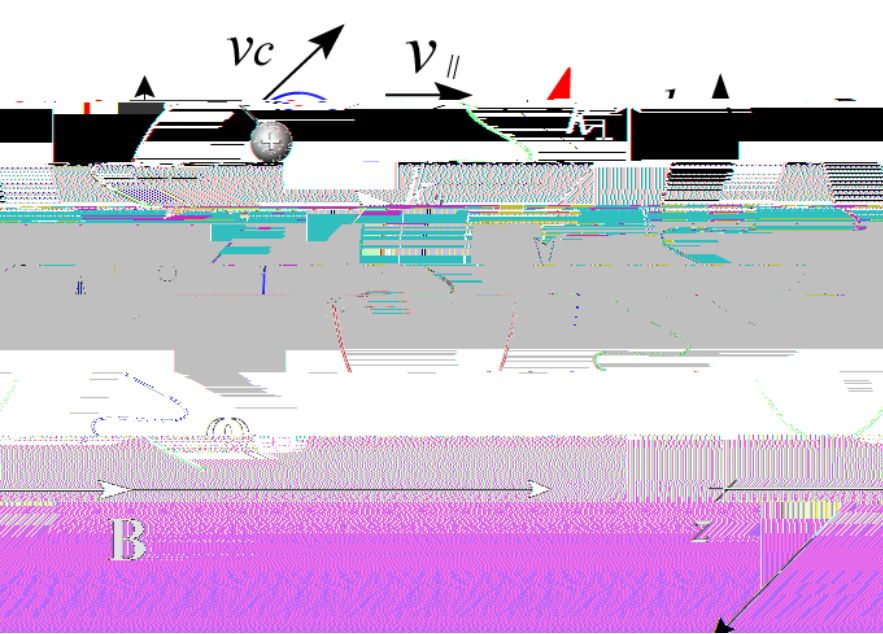


$$\text{Average current } I [\text{A}] = e [C] v_{\parallel} [\text{m/s}] \int e \langle v_{\parallel} \rangle$$



$$v_{\text{eff}} = \frac{1}{\omega} \left(\frac{\partial f}{\partial t} \right) \approx \frac{E^2}{2m\omega}$$



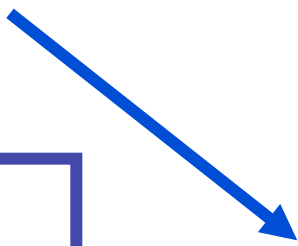


Resonance curves (h) $\omega = N\omega_c + kv_{||}$

$m_0 \omega_c < m_0 \omega_c$

$\omega_c^2 > \omega_c^2$

$\omega_c^2 > \omega_c^2$



$v \sin \theta = v_c$

ECRH

CD



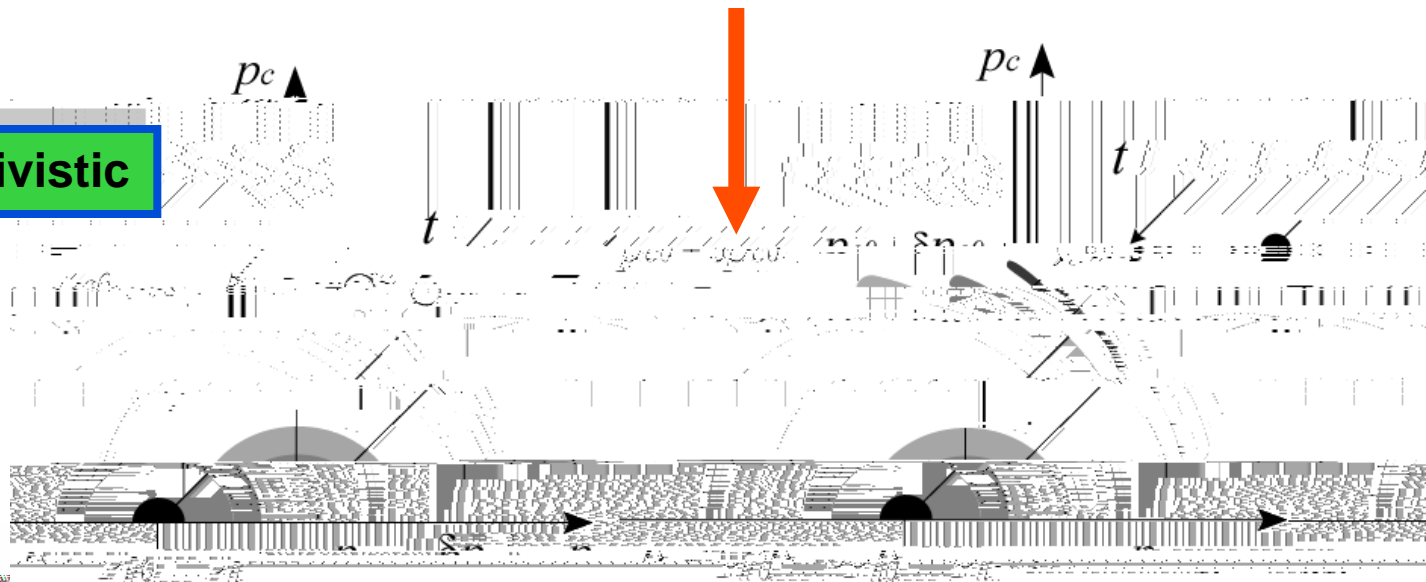
Current generation I : 2D response



1D classical



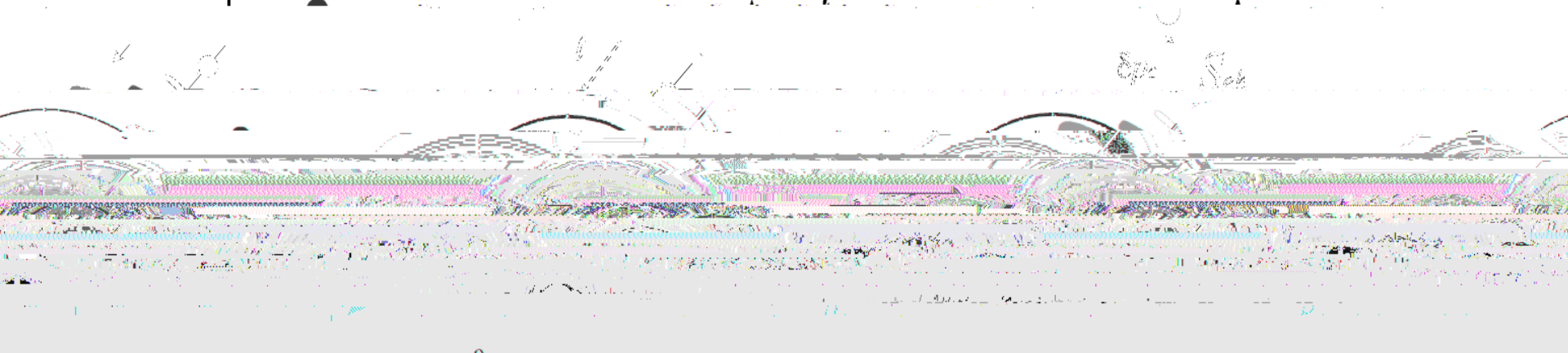
2D relativistic



2 D Model

$$\epsilon = \epsilon_0 \left(1 + \frac{4\pi n^2}{\omega^2} \right) = \epsilon_0 \left(1 + \frac{4\pi e^2 N}{m^2 \omega^2} \right)$$

$$p \sin\theta = p_c \uparrow$$



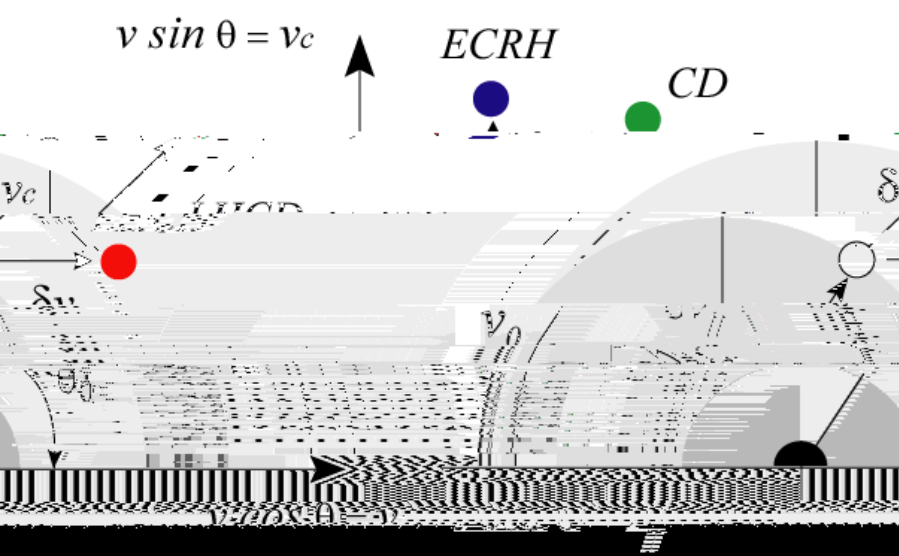
$$\delta\omega = \hbar k_{\parallel} v_{\parallel}$$

$$\gamma\omega - k_{\parallel} \gamma v_{\parallel} = n\omega_c \longrightarrow$$

$$\delta p_{\parallel} = \hbar k_{\parallel}$$

$$n\hbar\omega$$





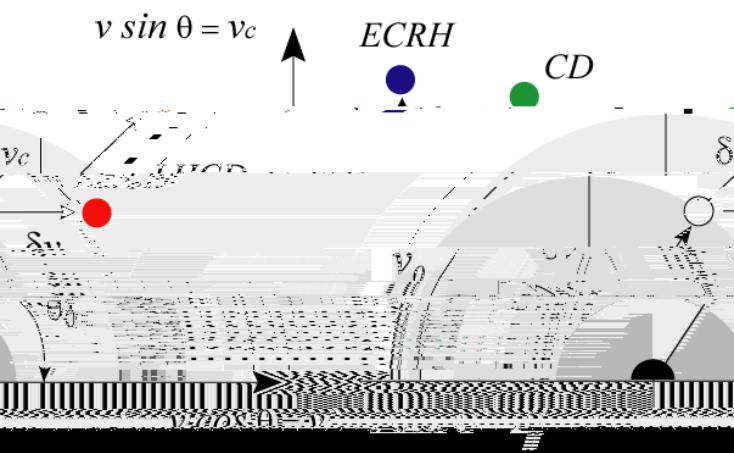
Photon absorption

$$S_{\text{ph}} = \frac{\partial}{\partial t} \int_V n_e n_p dV = \frac{\partial}{\partial t} \int_V n_e n_p dV$$

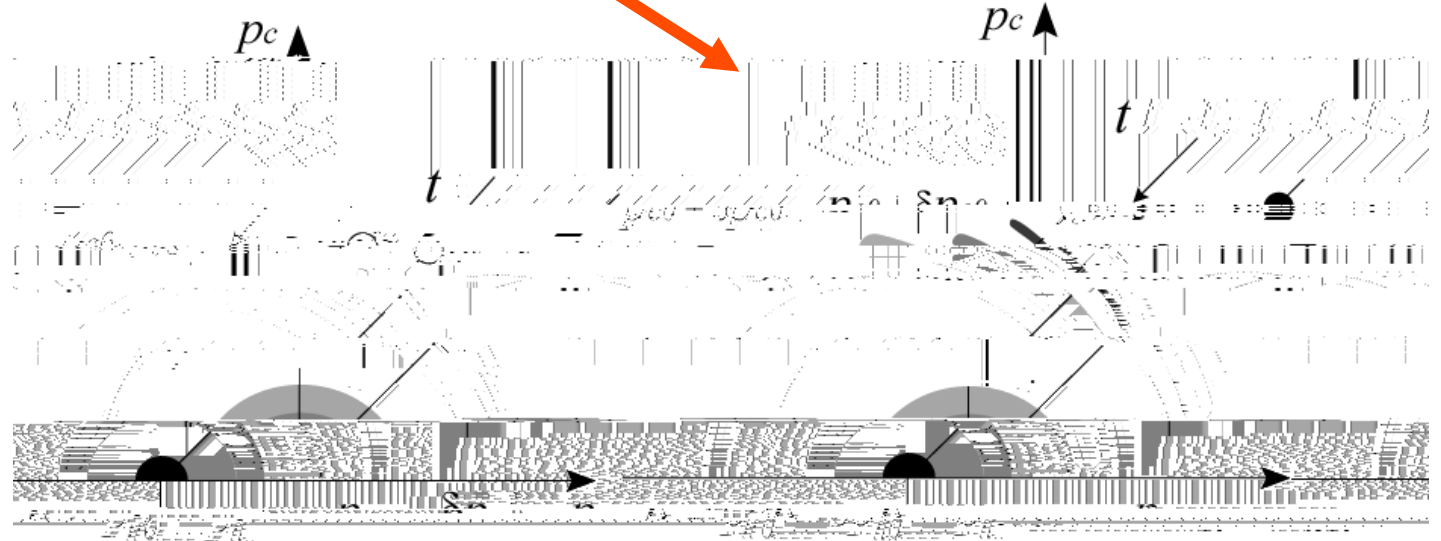
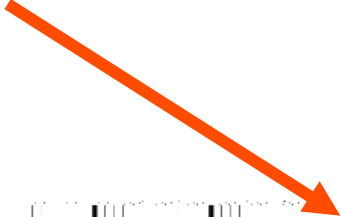
$$I_{\text{ph}}(\omega) = \frac{1}{4\pi} \int_V \frac{\partial}{\partial t} \left[\frac{1}{\omega} \frac{\partial}{\partial t} \left(\frac{1}{\omega} \frac{\partial}{\partial t} \right) \right] S_{\text{ph}} dV$$

- 1. $\frac{\partial}{\partial t} \left(\frac{1}{\omega} \frac{\partial}{\partial t} \right) S_{\text{ph}}$
- 2. $\frac{\partial}{\partial t} \left(\frac{1}{\omega} \frac{\partial}{\partial t} \right) S_{\text{ph}}$
- 3. $\frac{\partial}{\partial t} \left(\frac{1}{\omega} \frac{\partial}{\partial t} \right) S_{\text{ph}}$
- 4. $\frac{\partial}{\partial t} \left(\frac{1}{\omega} \frac{\partial}{\partial t} \right) S_{\text{ph}}$
- 5. $\frac{\partial}{\partial t} \left(\frac{1}{\omega} \frac{\partial}{\partial t} \right) S_{\text{ph}}$
- 6. $\frac{\partial}{\partial t} \left(\frac{1}{\omega} \frac{\partial}{\partial t} \right) S_{\text{ph}}$
- 7. $\frac{\partial}{\partial t} \left(\frac{1}{\omega} \frac{\partial}{\partial t} \right) S_{\text{ph}}$
- 8. $\frac{\partial}{\partial t} \left(\frac{1}{\omega} \frac{\partial}{\partial t} \right) S_{\text{ph}}$





Electron-hole collisional relaxation



∂f $\delta U(p_0)$

Evolution

Collisions

Photons Absorption



Excitation - Relaxation

$$\delta(n - n_c - \delta n_c) = \delta(n - n_c)$$

$$p \sin \theta = p_c \uparrow$$



$$\frac{1}{\omega} \frac{\partial}{\partial t} \left(\frac{1}{\omega} \frac{\partial}{\partial t} \left(\frac{1}{\omega} \frac{\partial}{\partial t} \left(\frac{1}{\omega} \frac{\partial}{\partial t} \right) \right) \right) = \dots$$



Excitation - Relaxation

$$\delta H(k_{||}, \omega) = N \hbar \omega_{||} \delta f$$

$$\exp[Ct] = 1 + Ct + \frac{Ct \cdot Ct}{2!} + \frac{Ct \cdot Ct \cdot Ct}{3!} + \dots$$

$$\exp[Ct] = 1 + Ct + \frac{Ct \cdot Ct}{2!} + \frac{Ct \cdot Ct \cdot Ct}{3!} + \dots$$



$$f(n, n, t, t) = A(t, t) \exp[C(n)(t, t)] H(k_{||}, \omega) S(n) \delta(n, n)$$

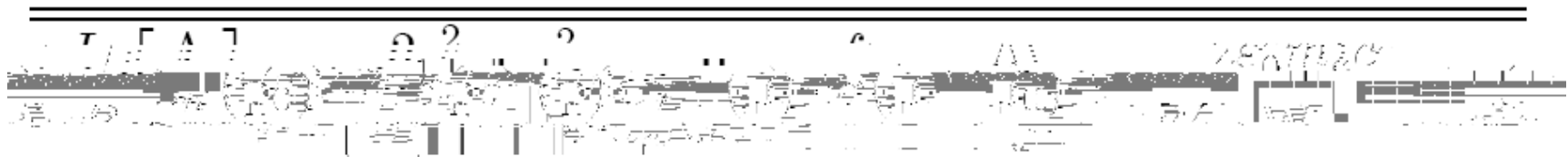


$$I_{\text{Thomson}} = I_{\text{A}} = e [C] v_{\parallel} \langle v_{\parallel} \rangle$$

$$I_{\text{Thomson}} = e [C] v_{\parallel} \langle v_{\parallel} \rangle = e [C] v_{\parallel} \int_{-\infty}^{\infty} v_{\parallel} f(v_{\parallel}) dv_{\parallel} = e [C] v_{\parallel} \int_{-\infty}^{\infty} v_{\parallel}^2 f(v_{\parallel}) dv_{\parallel} = e [C] v_{\parallel} \int_{-\infty}^{\infty} v_{\parallel}^2 f(v_{\parallel}) dv_{\parallel}$$

$$\delta U(k_{\parallel}, \omega) = W(k_{\parallel}, \omega) \delta t$$

$$W(k_{\parallel}, \omega) = \frac{1}{L} \int_{-\infty}^{\infty} v_{\parallel}^2 f(v_{\parallel}) dv_{\parallel}$$

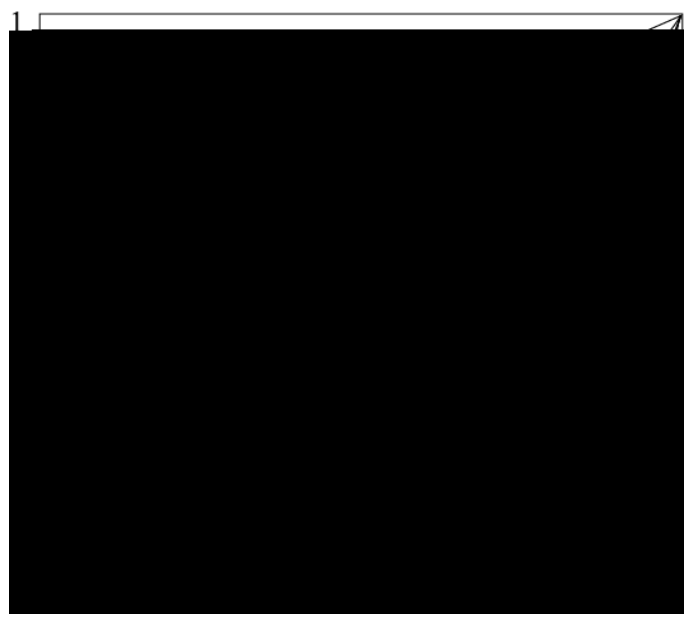


$$\frac{\partial}{\partial t} \left(\frac{1}{L} \int_{-\infty}^{\infty} v_{\parallel}^2 f(v_{\parallel}) dv_{\parallel} \right) = \frac{\partial}{\partial t} \left(\frac{1}{L} \int_{-\infty}^{\infty} v_{\parallel}^2 f(v_{\parallel}) dv_{\parallel} \right)$$

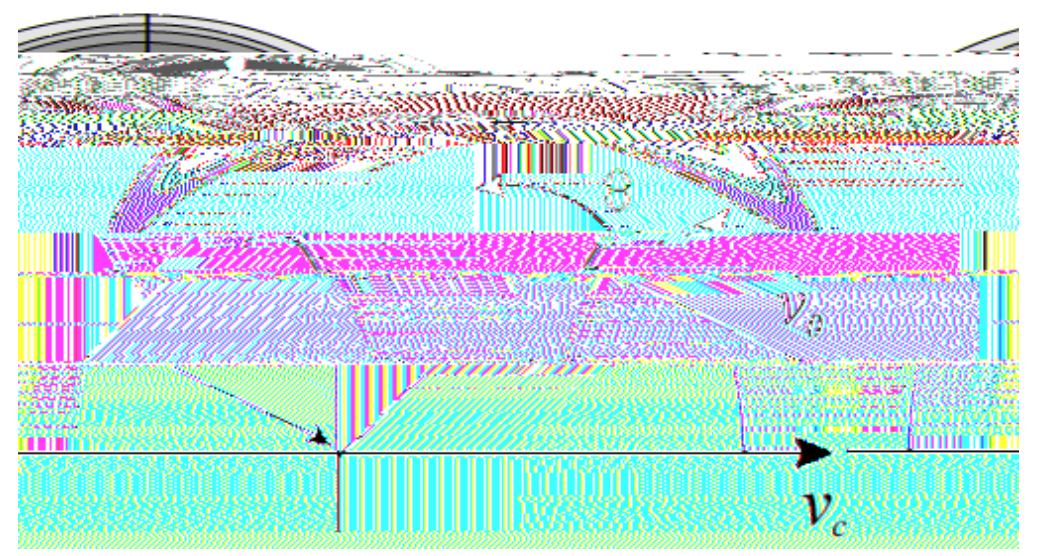


$l=0$ $l=1$ $l=2$ $l=3$ $l=4$ $l=5$ $l=6$ $l=7$ $l=8$ $l=9$ $l=10$

$$l=+\infty \quad (2l+1) \Gamma_m(2l+1) \quad \frac{(Z+1)l(l+1)}{2}$$



v_{θ} \blacktriangle



$$l=+\infty \quad (2l+1) \Gamma_{\infty} (2l+1) \quad \frac{(Z+1)l(l+1)}{2}$$

$$\left[k_{\parallel} \partial_t + n\omega_c - 1 \partial_t \right]$$

Handwritten notes in pink:
 - ∂_t \rightarrow ∂_t
 - ∂_t \rightarrow ∂_t
 - ∂_t \rightarrow ∂_t

$$2\varepsilon_0^2 m_e c^2 \quad [A] \quad [10^{20} \text{m}^{-3}]$$

Handwritten notes in green:
 - ε_0 \rightarrow ε_0
 - m_e \rightarrow m_e
 - c^2 \rightarrow c^2
 - $[A]$ \rightarrow $[A]$
 - $[10^{20} \text{m}^{-3}]$ \rightarrow $[10^{20} \text{m}^{-3}]$



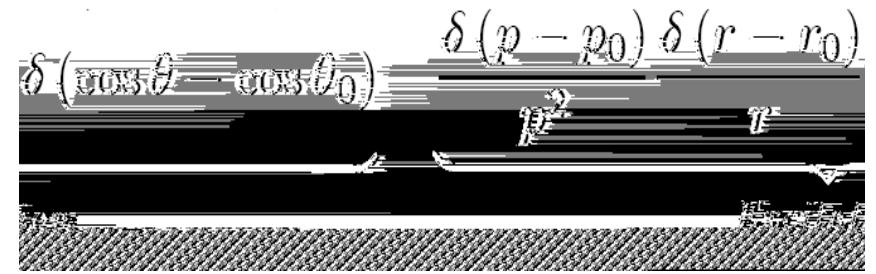
RF Current Transport

$$\epsilon = r_e = \frac{4\pi e^2 \omega_0^2}{\omega^2} \frac{1}{\epsilon} \frac{2}{\omega_e^2} \frac{3}{\omega^2} \frac{1}{\epsilon} \frac{4}{\omega^2} \frac{1}{\epsilon} \frac{1}{\omega^2}$$

RF current transport in a plasma

Collisions

Transport radial



$$l = +\infty$$

$$/ r p_0 \backslash$$

$$J_0(kr) J_0(kr_0)$$

$$e^{-\gamma(r-r_0)}$$

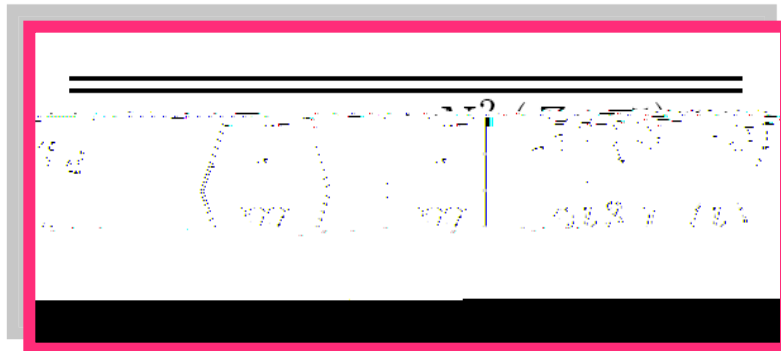


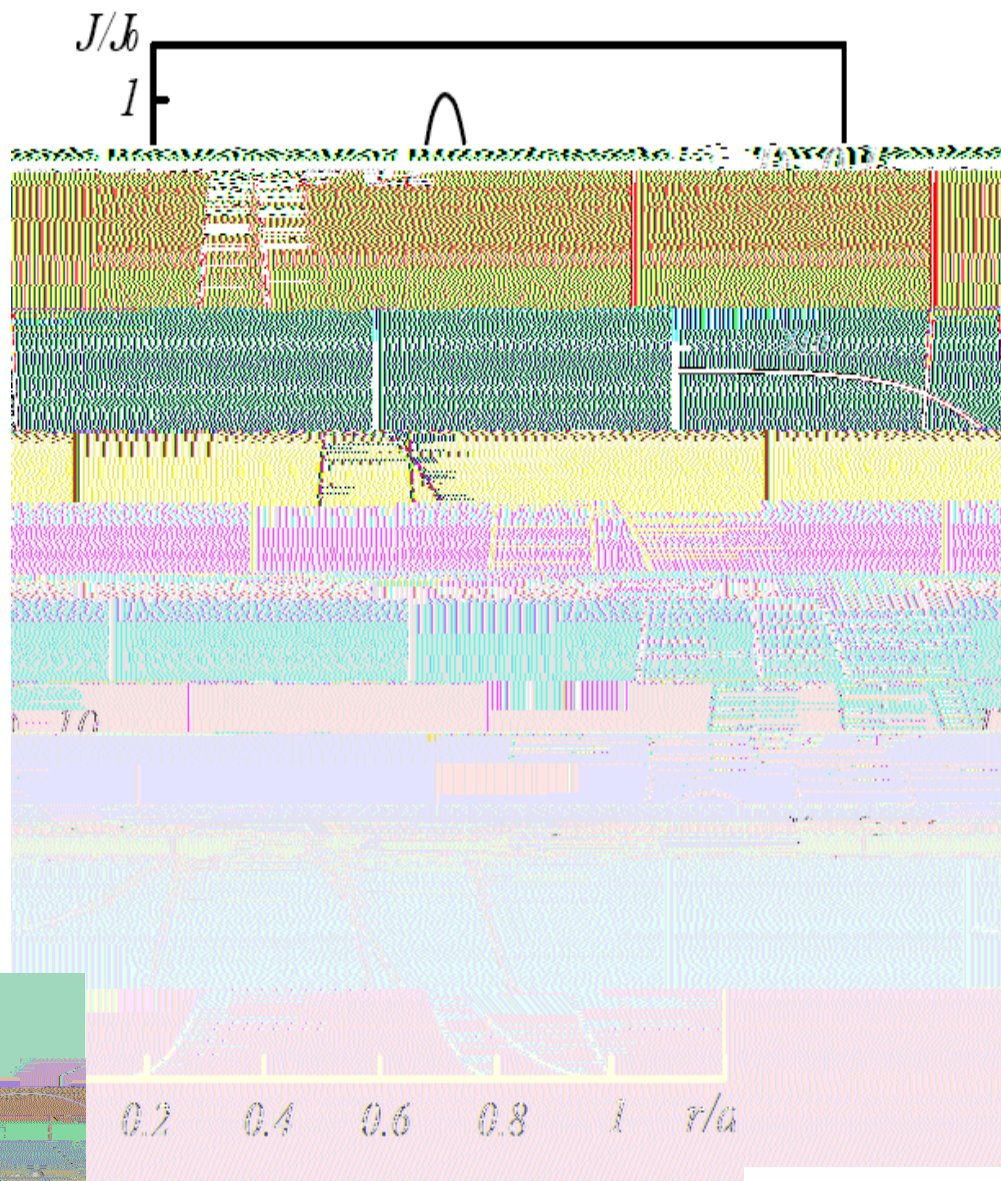
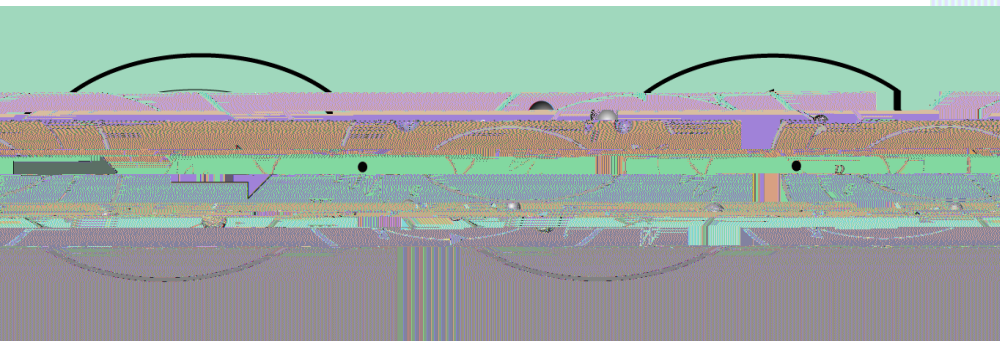
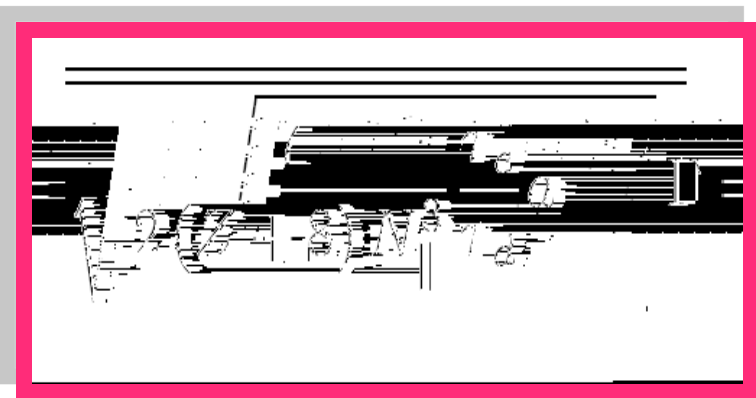
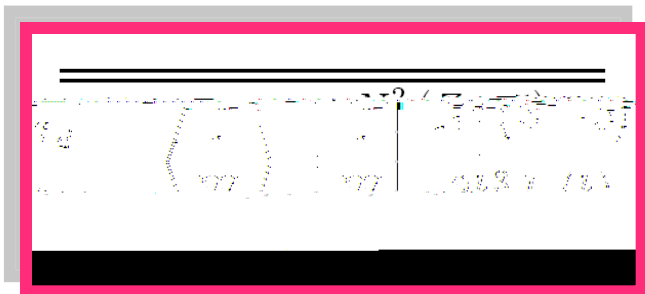
$$I(r) = \frac{1}{r} \left[1 - \frac{\partial}{\partial r} \left(\sin^2 \theta_0 \frac{\partial}{\partial r} \right) \right] f$$

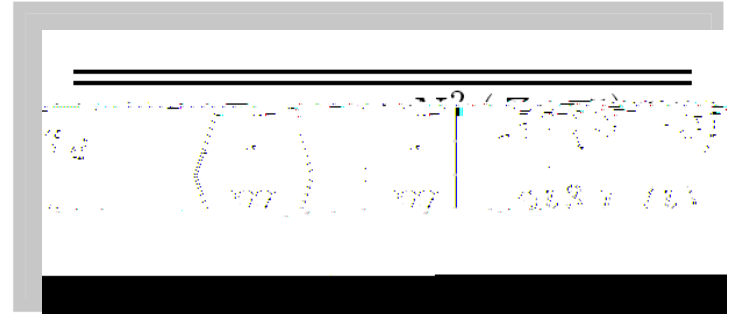
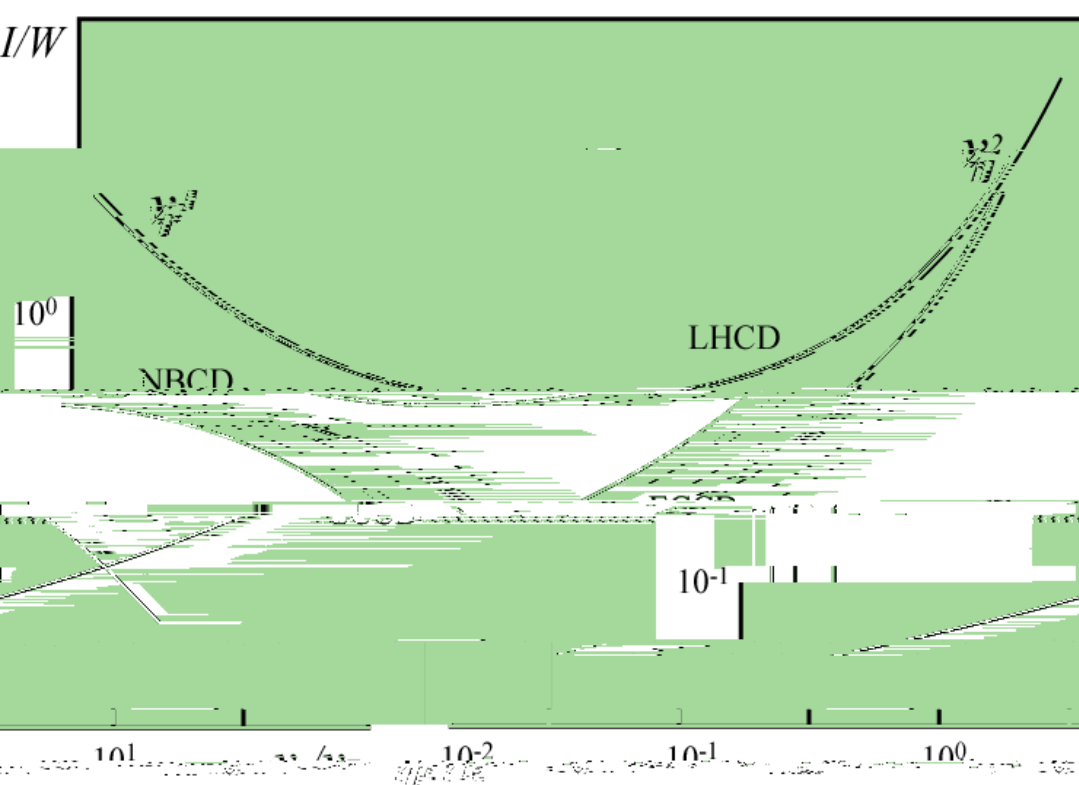
$$I(r) = \frac{1}{r} \left[1 - \frac{\partial}{\partial r} \left(\sin^2 \theta_0 \frac{\partial}{\partial r} \right) \right] f$$

$$J_0(k) = 0$$

$$D = \frac{a^2}{\tau_d}$$







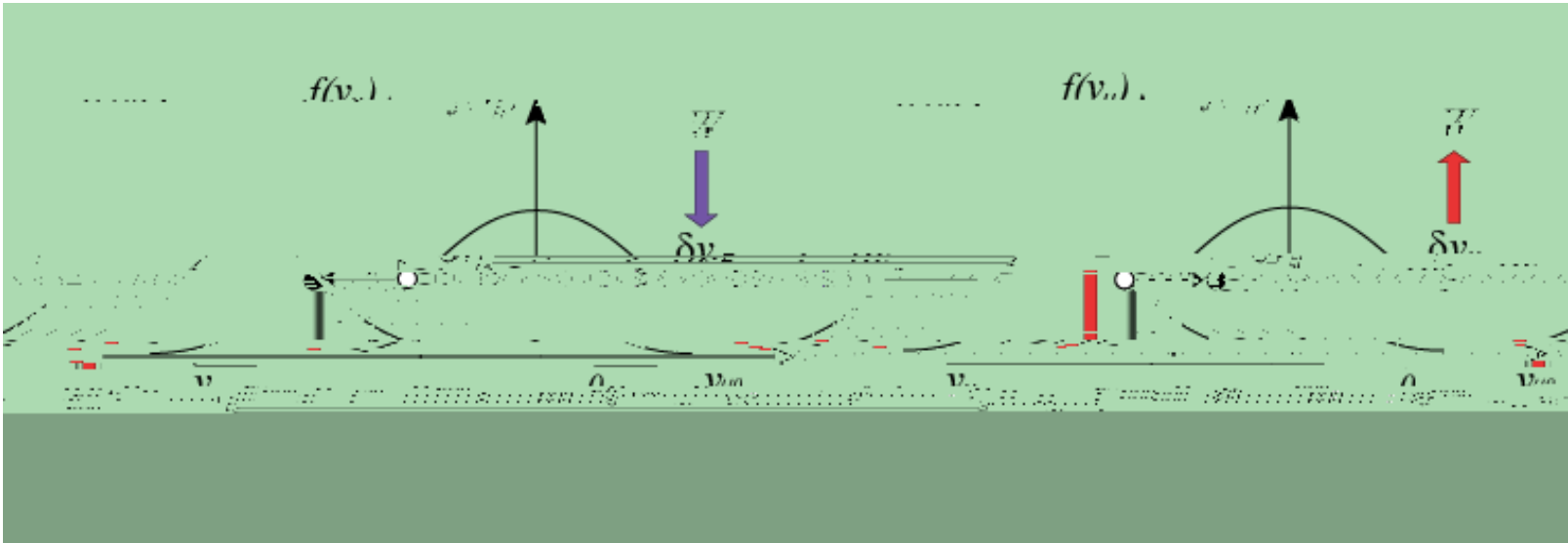
$$I \propto [A] \frac{2\epsilon_0^2 m_e c^2}{4N_{\parallel}^{-2}}$$

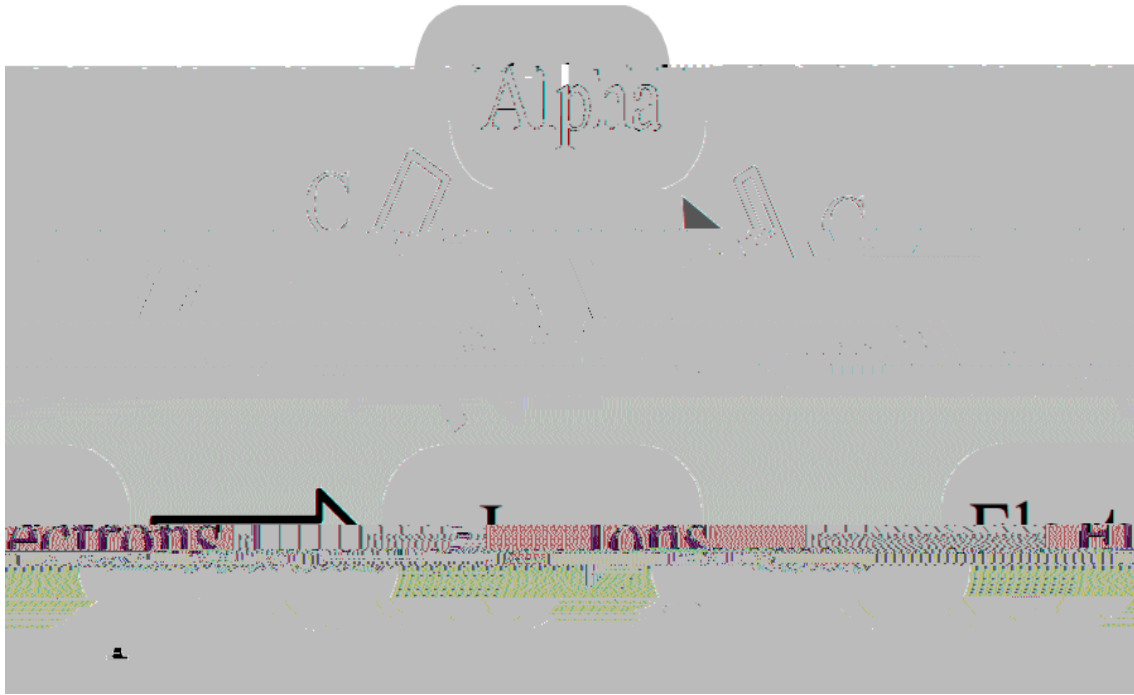
$$I \propto [A] \frac{2\epsilon_0^2 m_e c^2}{3N_{\parallel}^{-2}}$$



Free Energy Extraction

Figure 10.10: Free energy extraction from a nonequilibrium velocity distribution



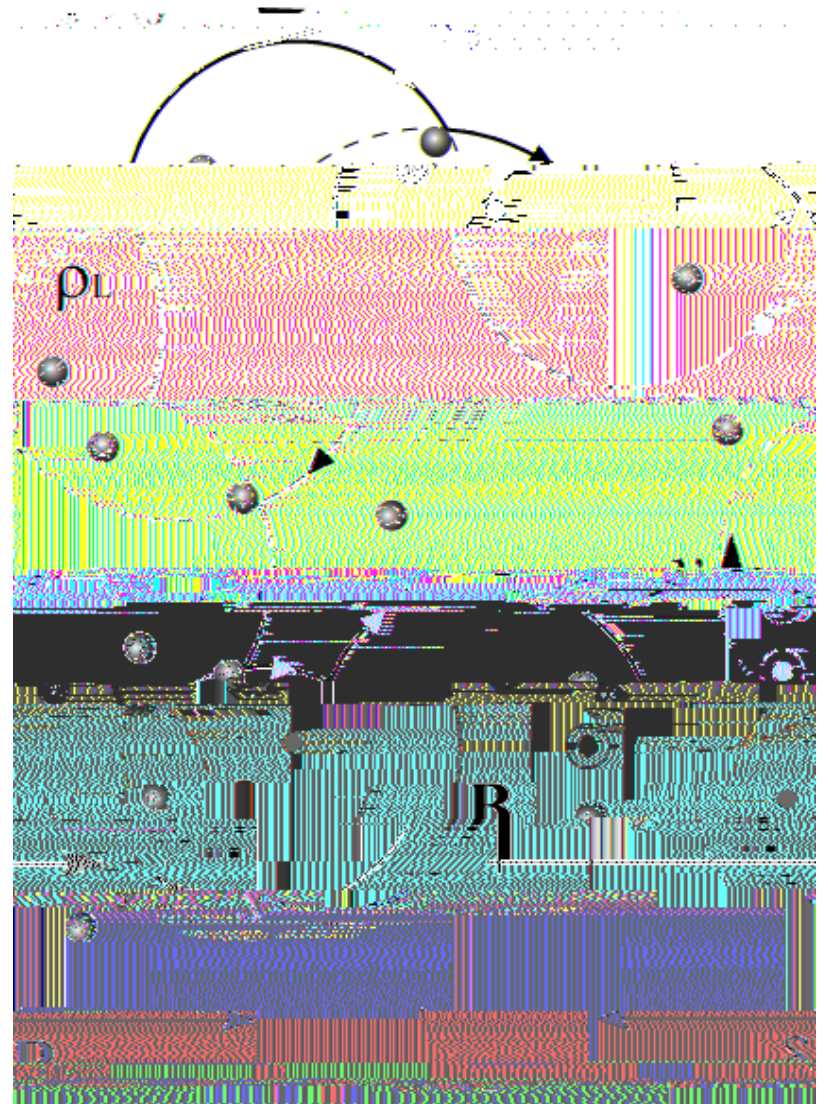




$$\langle \delta v^2 \rangle + \langle \delta v^2 \rangle$$



$$\langle \delta v^2 \rangle + \langle \delta v^2 \rangle$$



$$d\mathbf{v} = \frac{q}{m} \mathbf{D} + \nabla \phi - \mathbf{v} \times \mathbf{B}$$



17

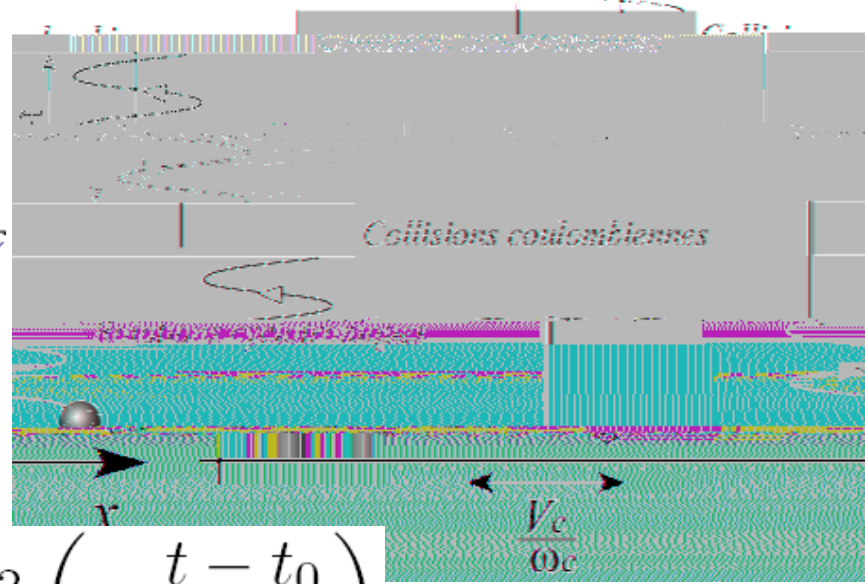
$t \uparrow$

clotroniques

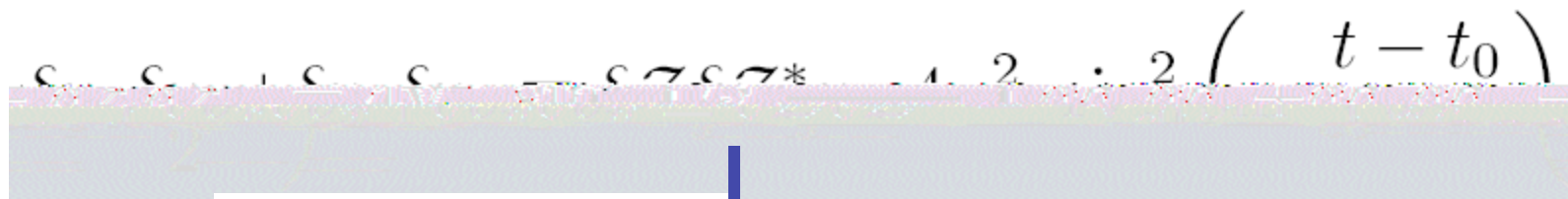
Rotations cy



$$\frac{1}{\epsilon_0} \left(\frac{d^2}{dt^2} + \omega_c^2 \right) \phi = -\rho$$



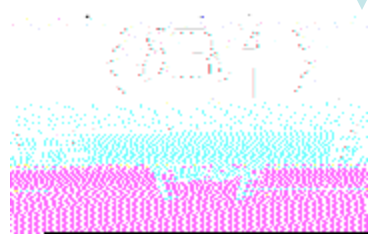
$$\left(\frac{d^2}{dt^2} + \omega_c^2 \right) \phi = -\rho$$



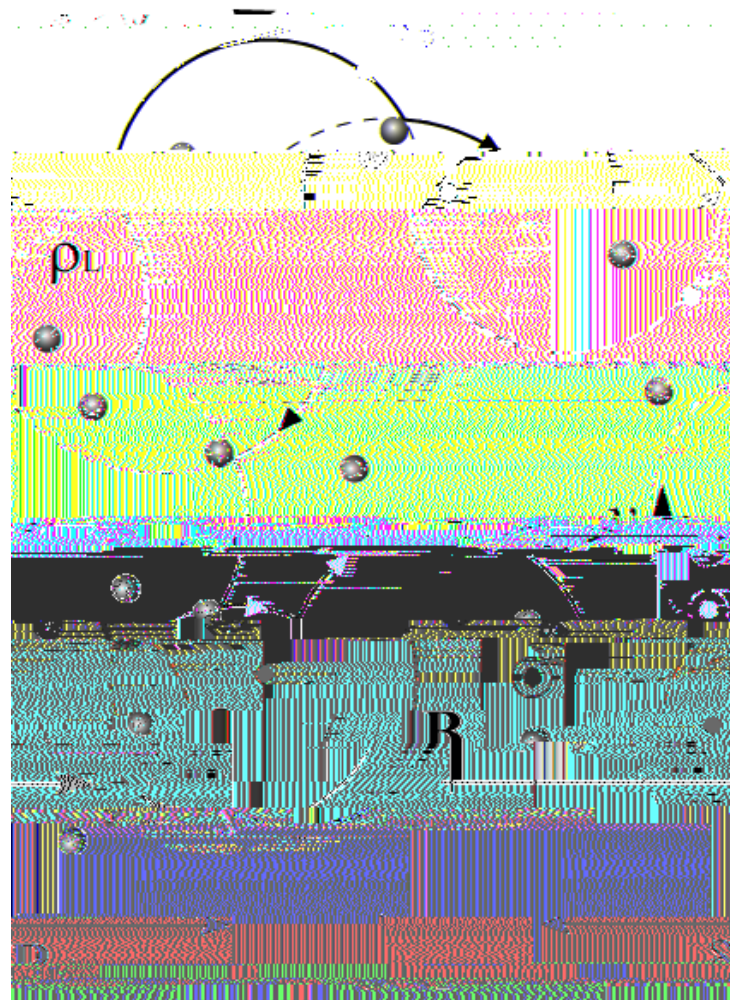
$$\langle \delta v^2 \rangle + \langle \delta v'^2 \rangle$$

$$\frac{1}{2} \left(\frac{1}{v^2} + \frac{1}{v_c^2} \right) \int_0^\infty \exp(-\nu \tau) d\tau$$

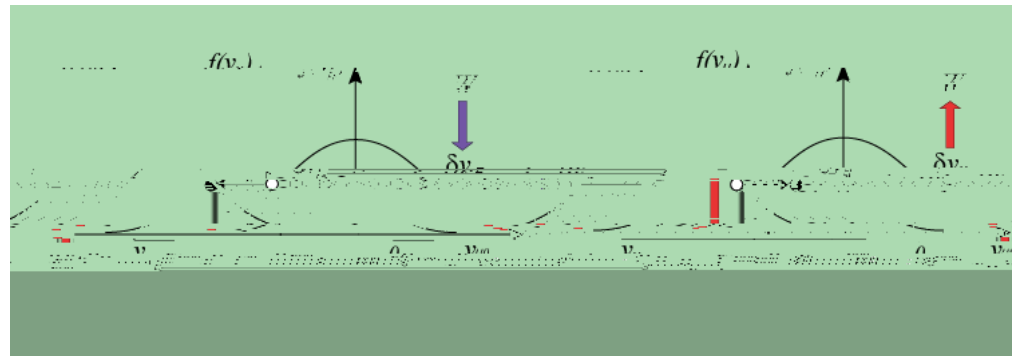
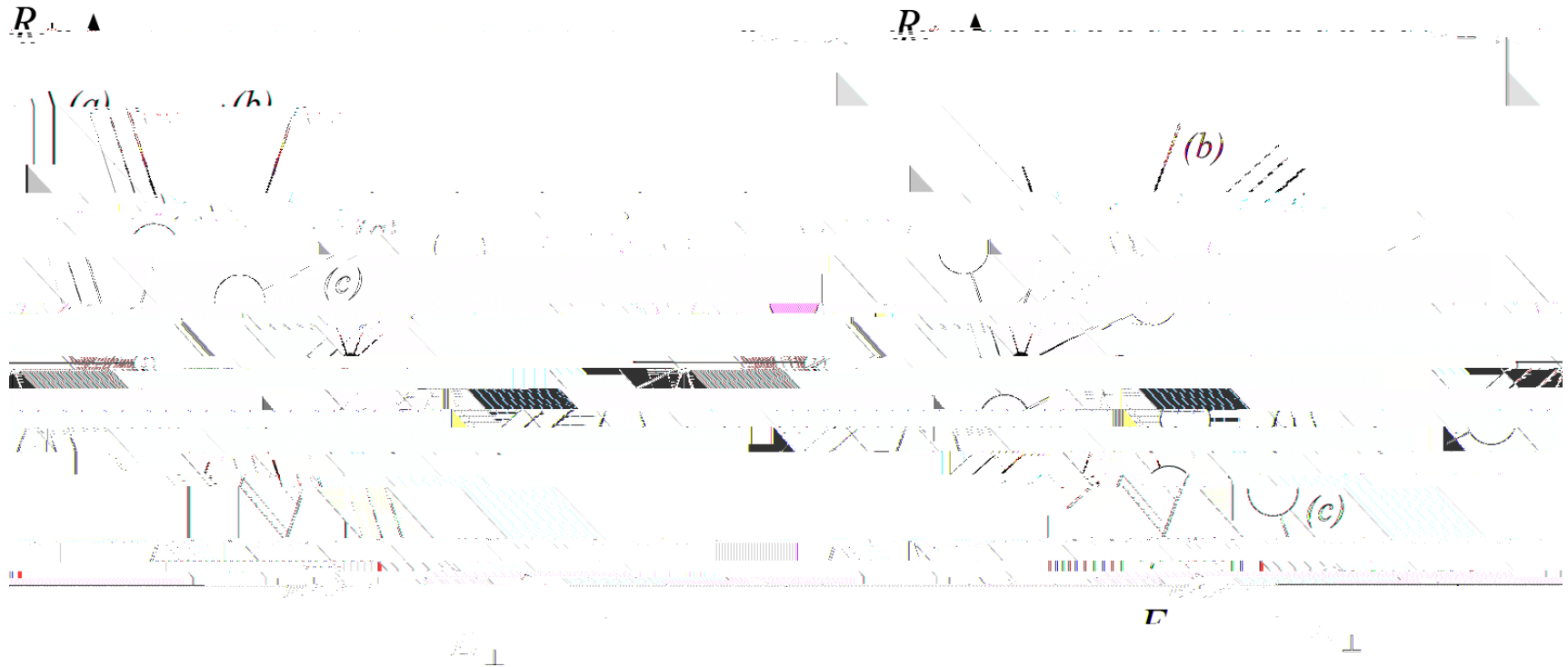
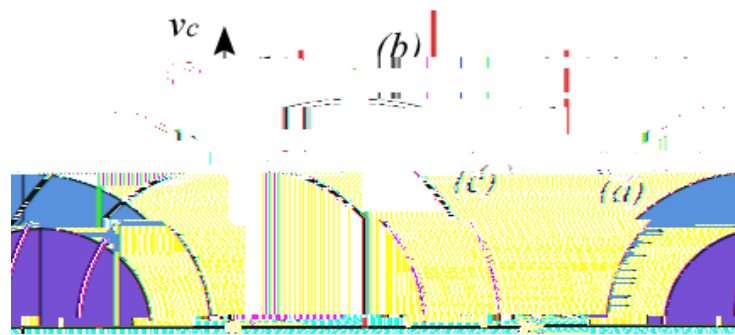
$$dP(\tau) = \nu \exp(-\nu \tau) d\tau$$



$$2v_c^2 \frac{\nu}{\nu^2 + \omega^2}$$



$$\langle \delta R_x^2 \rangle \quad \langle \delta R_y^2 \rangle \quad D_{\parallel}$$





for $\Omega \ll \omega$, $\Gamma_1 \approx \Gamma_2 \approx \Gamma_0$, $\Gamma_1 \approx \Gamma_2 \approx \Gamma_0$



$$\langle \delta v_c^2 \rangle = \frac{\pi e^2}{\omega^2} \dots$$

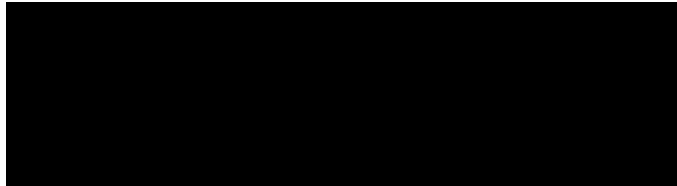
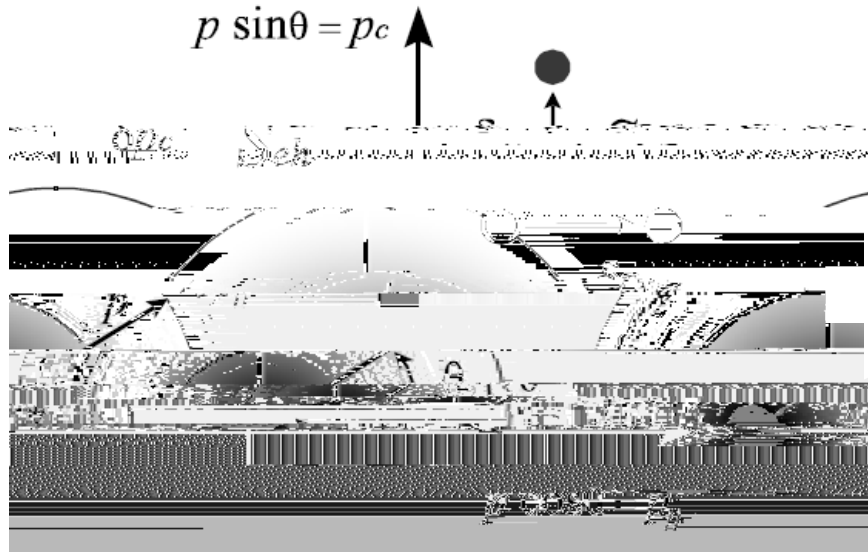


Physics of Landau and Cyclotron Resonances :

- Active and reactive power
- Plasma resonances
- Resonant interaction
- Random phase approximation RPA
- Quasi linear equation
- Landau absorption
- Cyclotron absorption
- Current generation 1D
- Current generation 2D
- Free energy extraction



Transfert d'impulsion



Samstag, 11. März 2011

$$n_e Z e^4 \Lambda$$

$$I \text{ [A]}$$

$$2 \epsilon_0^2 m_e$$

..2

$$n_e Z e^4 \Lambda$$

$$I \text{ [A]}$$

$$2 \epsilon_0^2 m_e$$

..3

